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Matlab Simulations of Dynamic Pricing on the Market of a Product Models

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ABSTRACT

This paper aims to illustrate the dynamic models of prices adjustment on a product market, such as Kaldor classic model and Extended Kaldor model with adaptive expectations, using simulations achieved by Matlab programming language. This way price trajectories will be plotted, using multiple sets of input data.

1. Introduction

In the systemic approach of real phenomena people are primarily seeking certain trajectories of the system, but there is a limited variety of commands that man can control.

The issue of system adjusting is to seize deviations from the target trajectory, to measure these deviations and to choose an override function to determine the return on the trajectory. This requires the construction of an adjustment function, which should associate for each time point and each state of the system, an input value, according to Bellman's principle: "Entries in the system are determined by current states."

2. Theoretical framework

2.1. Discrete-time linear models

As Kaldor models of prices adjustment are dynamic, linear and discrete models, we recall further, the main theoretical aspects related to these.

Discrete linear models are described by linear dynamic equations with finite differences of the first order and higher, and the time horizon is included in a discreet set:

$$\begin{cases} x_{t+1} = A_t x_t + B_t u_t \\ y_t = C_t x_t \end{cases}, t \in Z \qquad (2.1.1)$$

where x= state variables, u= input variables, y= output variables, and A,B,C= matrixes of coefficients.

a) First-order homogenous models

$$x_{t+1} = \alpha x_t \qquad (2.1.2)$$

$$x_t = C \cdot \alpha^t \qquad (2.1.3)$$

$$x_t = x_0 \cdot \alpha^t \qquad (2.1.4)$$

The general solution of the model, (2.1.3) is achieved in the case of not knowing the initial value of the variable, x_0, with C as a real constant and t∈{0,1,2,....}. But if the initial value of the state variable is known, the model has unique solution, (2.1.4).

Types of trajectories

The trajectories solutions of the model are exponential functions of time, that depend on α parameter values, as follows:

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1) if $|\alpha|<1: x_t \to 0$, the trajectory is amortized, monotonous in time, if α> 0, or oscillating in time, with improper oscillations caused by $(-1)^t$, with T = 2 periodicity, if α <0.

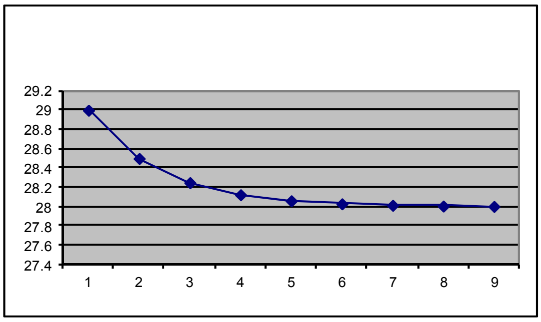

Fig.1.1. Monotonous amortized trajectory

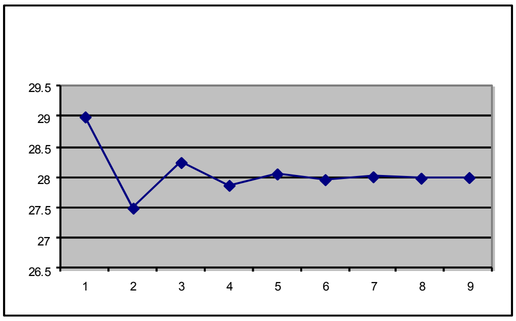

Fig.1.2.Oscillatory amortized trajectory

2) if $|\alpha|>1: x_t \to \infty$, the trajectory is explosive, monotonous in time if α> 0, or oscillating in time, with improper oscillations caused by $(-1)^t$, with T = 2 periodicity, if α <0.

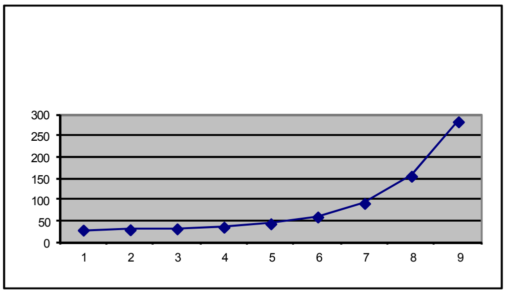

Fig.1.3. Monotonous explosive trajectory

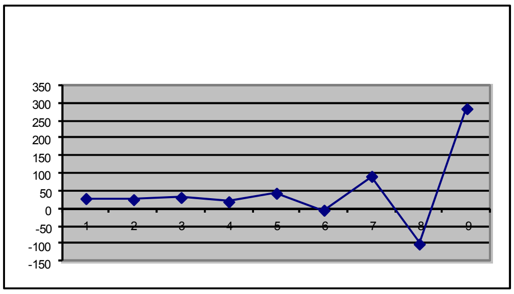

Fig.1.4. Oscillatory explosive trajectory

3) if $|\alpha|=1: x_t = \pm x_0$, the trajectory shall have a constant amplitude.

b) ***First-order non-homogeneous models***

$$x_{t+1} = \alpha x_t + \beta \qquad (2.1.5.)$$

The system solution will be, in this case:

$$\begin{aligned} x_t &= C \cdot \alpha^t + x_e, \alpha \neq 1 \\ x_t &= C + \beta t, \alpha = 1 \end{aligned} \qquad (2.1.6)$$

where x_e= equilibrium value, and if the initial conditions are known, the system unique solution can be determined:

$$\begin{aligned} x_t &= (x_0 - x_e) \cdot \alpha^t + x_e, \alpha \neq 1 \\ x_t &= x_0 + \beta t, \alpha = 1 \end{aligned} \qquad (2.1.7)$$

The convergence of this solution trajectories also depend on α, as follows: if |α|<1 the trajectories are monotonous or oscillatory amortized, and if |α|>1 the trajectories are monotonous or oscillatory explozive.

$$\begin{aligned} &1)|r|<1 \Rightarrow \lim_{t\to\infty} x_t = \lim_{t\to\infty}(C\alpha^t + \frac{\beta}{1-\alpha}) = \frac{\beta}{1-\alpha} = x_e \\ &2)|r|>1 \Rightarrow \lim_{t\to\infty} x_t = \lim_{t\to\infty}(C\alpha^t + \frac{\beta}{1-\alpha}) = \infty \end{aligned} \qquad (2.1.8)$$

2.2. The classic Kaldor model (dynamic pricing on the market of a product model)

We recall that the model is based on the following assumptions: demand depends on current prices, the current period supply depends on the previous period prices, and the market price level adjusts each period, depending on the deviation between supply and demand.

$D_t=\alpha+\beta p_t$ (2.2.1.)

$S_t=a+bp_{t-}$ (2.2.2.)

$D_t = S_t$ (2.2.3.)

If we find ourselves in normal market conditions, then $dD/dp=\beta <0$ (negative marginal demand) and $dS/dp=b>0$ (positive marginal supply).

Static equilibrium between supply and demand shall be considered to determine p^e, the equilibrium price : $\alpha+\beta p^e= a+bp^e$, resulting $p^e=\frac{\alpha-a}{b-\beta}>0 \Rightarrow \alpha-a>0$.

The dynamic equation of state variable, ie the price, is deducted from the dynamic demand-supply equilibrium: $\alpha+\beta p_t= a+bp_{t-1}$

$$\Rightarrow p_t=\frac{b}{\beta}p_{t-1}+\frac{a-\alpha}{\beta} \qquad (2.2.4.)$$

According to the theory of discrete linear models, the general solution is of the form: $p_t=(\frac{b}{\beta})^t C+p^e$ (2.2.5.)

where the constant C is determined based on the initial value p_0.

$$p_t=(\frac{b}{\beta})^t(p_0-p^e)+p^e \qquad (2.2.6.)$$

The stability condition is: $\left|\frac{b}{\beta}\right|<1$ so that $p_t \rightarrow p^e$.

2.3. Extended Kaldor model with adaptive expectations

For this extended model, the second hypothesis is replaced by the assumption that producers anticipate that the price level at time t will be p_t^a.

$D_t=\alpha+\beta p_t$ (2.3.1.)

$S_t=a+bp_t^a$ (2.3.2.)

$D_t = S_t$ (2.3.3.)

$p_t^a=p_{t-1}+c(P_N-p_{t-1})$ (2.3.4.)

where $c\in(0,1)$ is called the delay in reaching the normal price P_N.

The equilibrium price is obtained from the static equilibrium between supply and demand:

$$p^e=\frac{bcP_N+a-\alpha}{\beta-b+cb}=\frac{b\,c}{\beta-b+cb}P_N-\frac{\alpha-a}{\beta-b+cb} \qquad (2.3.5.)$$

Dynamic equation of price is deducted from dynamic equilibrium $D_t = S_t$:

$$p_t=(\frac{b}{\beta}-\frac{bc}{\beta})p_{t-1}+\frac{bc}{\beta}P_N-\frac{\alpha-a}{\beta} \qquad (2.3.6.)$$

We know that the general trajectory will be:

$$p_t = [\frac{b}{\beta}(1-c)]^t C + p^e \qquad (2.3.7.)$$

where the constant C is determined based on the initial value, for t=0.

If p_0 is known, the unique trajectory will be:

$$p_t = [\frac{b}{\beta}(1-c)]^t (p_0 - p^e) + p^e \qquad (2.3.8.)$$

The stability condition in this model with adaptive expectations will be: $\left|\frac{b_1}{b}(1-c)\right| < 1$. But $\left|\frac{b_1}{b}(1-c)\right| < \left|\frac{b}{b_1}\right| < 1$, and hence adaptive expectations model has a higher speed for reaching equilibrium than the classic model.

3. Plotting the price evolution trajectories in Matlab

The price adjustment mechanism to balance supply and demand can be seen in the table below, based on which was accomplished the Matlab source code as well, for plotting evolution trajectories.

Tab.2.1. Price evolution

T	0	1	2	...n-1	n
p_t	P_0	P_1	P_2	...p_{n-1}	p_n
D_t	D_0	D_1	D_2	...D_{n-1}	D_n
S_{t+1}	S_1	S_2	S_3	...S_n	

Arrows between p_t and S_{t+1} show the producer's decision and those between Dt and p_t indicate the price formation.

We will further simulate all the possible trajectories for the price, using both classical, and the adaptive expectations models.

Thus, in the following graph (3.1.) it can be observed an oscillatory explosive trajectory obtained on the classical model for these inputs: α=98 ,β=-1 , a=48 , b=1.2 , p_0=25 and a time interval of t=15.

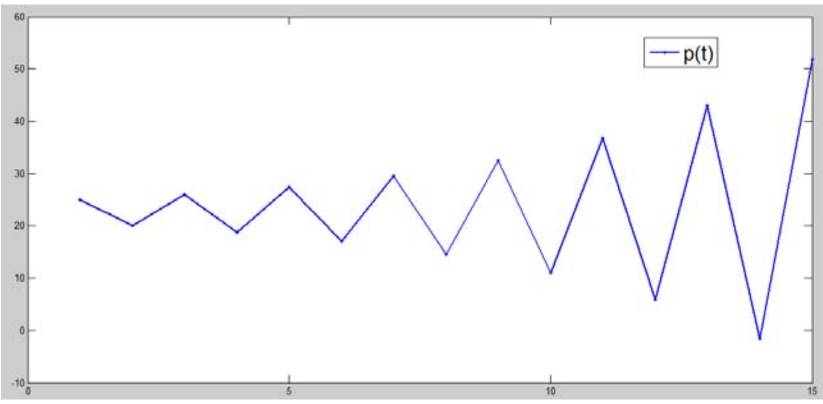

Fig.3.1. Price oscillatory explosive trajectory for Kaldor model

In this case, when t tends to t_final price is exponentially moving away from its equilibrium level.

In the following graph (3.2.) it can be observed an oscillatory amortized trajectory obtained on the classical model for the following inputs: α=78 ,β=-0,75 , a=-1, b=0,5, p0=40 and a time interval of t=20.

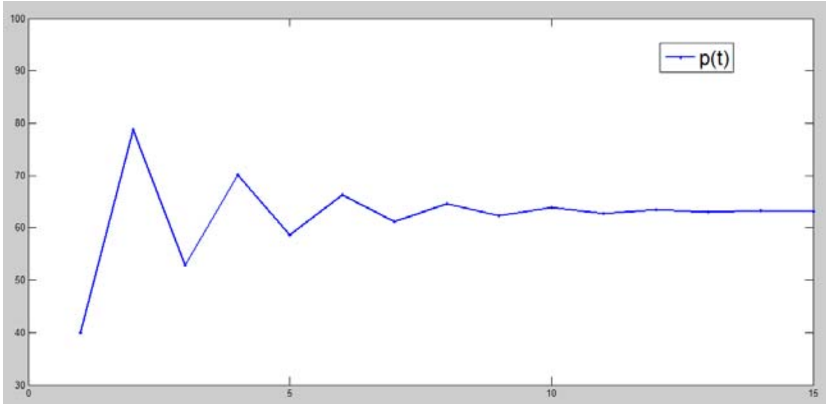

Fig.3.2. Price oscillatory amortized trajectory for Kaldor model

In this case, the trajectory is stabilizing beginning with time moment t = 12.

If we keep the following inputs: α=78 ,β=-0.75, a=-1, p0=40 and time interval t=20, but we make b=0,75, such that the ratio b/β become 1, then we get a constant amplitude oscillating trajectory in 3.3. graph.

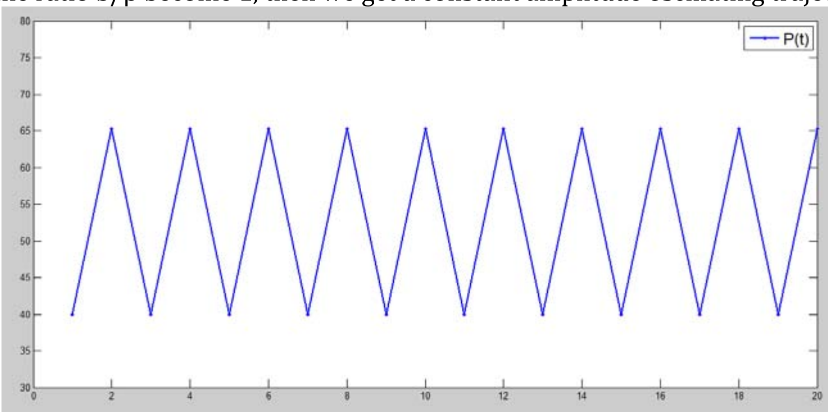

Fig.3.3. Price oscillatory trajectory with constant amplitude, for Kaldor model

In this case the price is not moving away, nor is approaching the equilibrium level over time.

An oscillatory explosive trajectory can be observed in the following graph (3.4.), obtained on the extended adaptive expectations model for the following inputs: α=98, β=-1, a=48, b=1.2, c=0,1 , p0=25 , P_N=26 and a time interval of t=15.

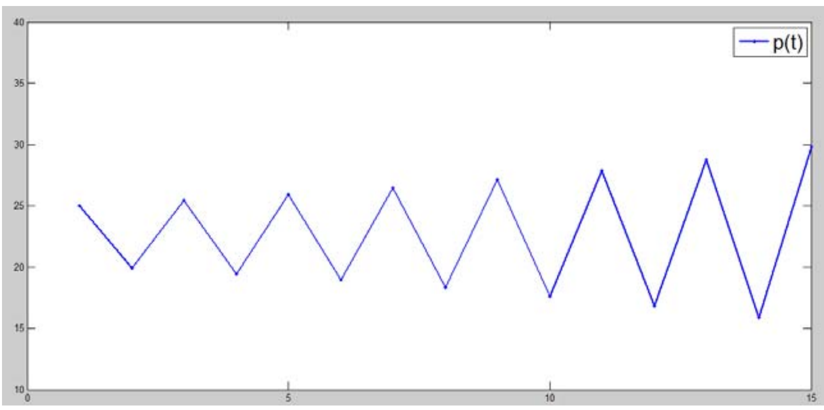

Fig.3.4. Price oscillatory explosive trajectory for the Extended Kaldor model

In the following graph (3.5.) it can be observed an oscillatory amortized trajectory obtained on the extended model for the following inputs: α=78, β=-0,75, a=-1, b=0,5, c=0,3, p_0=40, P_N=45 and a time interval of t=15.

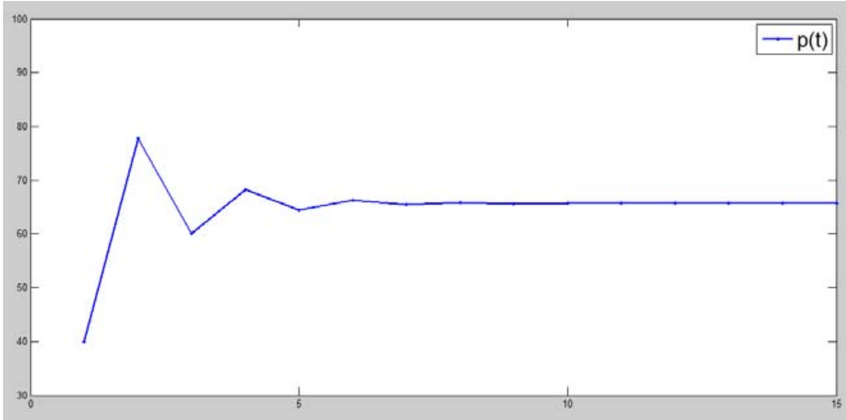

Fig.3.5. Price oscillatory amortized trajectory for the Extended Kaldor model

Although we have been used the same coefficients for demand and supply functions like in 3.2. graph, as opposed to the classical model, in this case, convergence to equilibrium level was reached at t = 9.

5. Conclusions

Choosing appropriate inputs, we could trace all kinds of evolution trajectories for the two models, the classic and the extended one. If graphs 3.2. and 3.5. are compared, it can be observed that in both cases the price tends to equilibrium level, but in the second case of adaptive expectations, convergence is achieved with greater speed. In real markets occur more complex processes, and price evolution trajectory is influenced by a variety of factors difficult to quantify. According to Rizescu, Zamfir and Enache(2010), access to information is a necessity that had become more stringent in the actual economic context, as decisions must be taken in the shortest time. Kaldor pricing models are useful to explain price movements based on lags between supply and demand decisions.

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