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HANKEL-NORM APPROXIMATION METHOD APPLIED FOR HIGH ORDER MODEL REDUCTION TO ROBUST CONTROLLER SYNTHESIS

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Abstract: In this paper, is presented an application of the Hankel–norm approximation method for model order reduction, to obtain a lower order model with the same performance. In order to show the effectiveness of the Hankel-methodologie an illustrative example is considered. Based on the background theory, the algorithm was applied on a real-life plant consist of a waste heat recovery for thermal water supply. The final aim of this study is a possible synthesis of a robust controller for the (uncertain) industrial-heat recovery processes.

Keywords: High order model, model reduction techniques, Hankel-norm approximation method.

1. INTRODUCTION

From a practical point of view, the responsibility of a control system designer is to build a system that will perform well under real-life conditions and constraints. Therefore, the designed control system must ensure stability and a high performance level in the presence of uncertainties (real environment may change with time, parameters can vary, unmodelled dynamics, etc.).

In standard textbooks, classical frequency domain control design techniques have as design specifications only the gain margin and phase margin. In control engineering, the property of a control system to operate properly under realistic conditions is referred to as *robustness*.

Historically speaking, the robustness issue was not strongly considered in the decades 1960s and 1970s. However, researchers as Horowitz, Houpis through their pioneering works, e.g. (Horowitz, 1963), have put a first milestone in the robustness field – the QFT (quantitative feedback theory). Step by step, researchers in analysis and design of multivariable (MIMO) control systems, such as A. J MacFarlane, H.H. Rosenbrock, Postletwhaite, (Rosenbrock, 1974; Postletwhaite et. al, 1981), extended the principles of classical control to the MIMO case in the 1970s. The robust control design concepts have been further developed in time, and in 1980s, by the work of Zames and Francis (Zames, 1981; Zames and Francis, 1983), begins the second milestone in the field of system robustness. Nowadays, the robust control procedures have evolved from the practical field, as initiated by the above pioneers and many others (e.g. Doyle, 1987; Doyle et. al, 1992; Safonov et. al, 1989; Chiang and Safonov, 1988). The results are the new paradigms which are systematic, efficient and elegant: H_2 control, H_∞ optimal control, loop shaping control, H_∞ optimization, μ-analysis and synthesis etc.

The robust controller resulting from any such procedure is a complex one and consequently, from practical reasons its order must be reduced. For a control system, both model reductions of initial orders (of the original plant model and of controller) obviously, are similar in procedure (Choi et. al, 1994;

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Gu et. al, 2005; Zhou et. al, 1996; Safonov and Chiang, 1989). In this paper, we use an illustrative example to show the methodology and its efficient model order reduction. Although simplistic, the example is a sufficient numerical support to show the procedure and its results.

The paper is organized as follows: the next section provides a description of the example employed throughout the exercise. The third section presents the theoretical background and motivations for developing model-order reduction techniques. The results are given and discussed in section four and some conclusions summarize the main outcome of this paper and point to some perspectives.

2. SYSTEM DESCRIPTION

The example proposed for this paper takes advantage of the re-use of industrial waste-heat. Metallurgical factories use high temperatures to melt and further process metals like aluminum, iron, cupper etc. These high temperatures can be used to heat up water, that later, can be sent to the cities to provide heating to residential areas. However, because of the remoteness of the factories, long pipelines have to be used normally without good thermal insulation, causing heating loses. The schematic representation of the system is depicted in figure 1. The hot water input flow Q is divided in two (or more) pipelines according to the parameter Alpha, and then it goes through 25 sections in each side. Each section has its own temperature because of the thermal losses. Finally the two pipelines join, through a mixing process, the output temperature of both sides.

From a practical point of view, this system is in fact more interesting with input the flow ratio regulation (α) for controlling the temperature at the end of the line. However, this implies a nonlinear model, which is far too complex for the illustrative purpose of this paper and makes the target of another future contribution.

Consider the distributed parameter system from figure 1. The energy balance for the first section is shown below:

$$(1)\quad \rho C_P V \dot{T}_{11} = \rho C_P Q_1 (T_{in} - T_{11}) - q_{11}$$

Where:

$q_{nm} = Heat\ flow \left[\frac{Joul}{s}\right] = External\ disturbances = Input$

$T_{in} = Input\ Temperature\ [°C] = input$

$T_{nm} = Distributed\ Temperature\ [°C] = States$

$T_{out\ 1}, T_{out\ 2} = Output\ temperatures\ [°C] = Ouputs$

$Q = input\ flow\ \left[\frac{m^3}{s}\right]$

$C_p = Specific\ Heat\ \left[\frac{Joul}{Kg°C}\right]$

$\rho = density\ \left[\frac{Kg}{m^3}\right]$

$\mathrm{V} = \mathrm{Volume}\ [\mathrm{m}^3]$

$\mathrm{V}_1 = \mathrm{Volume\ in\ the\ left\ side}[m^3]$

$\mathrm{V}_2 = \mathrm{Volumen\ in\ the\ right\ side}\ [\mathrm{m}^3]$

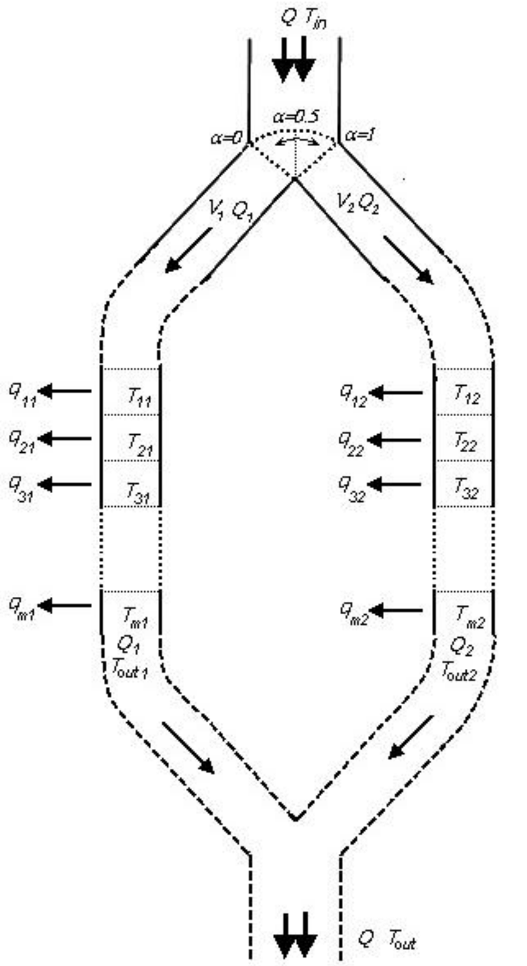

Fig.1. Schematic representation for the proposed system.

The ratio α is the relationship between the flows and volumes. It is defined in (2), and applied in (3), (4) and (5).

$$(2)\quad \alpha = \frac{Q_1}{Q}$$

$$(3)\quad 1 - \alpha = \frac{Q_2}{Q}$$

$$(4)\ V_1 = V\alpha$$

$$(5)\ V_2 = V(1-\alpha)$$

The full system can be represented as a set of differential equations, one for each section. They are presented from (6) until (13). Each one takes into account the temperature in the previous section, in order to compute the current section output. Finally, the total output of the system is computed in (14), as the mean value between the two output temperatures in each side.

$$(6)\ \dot{T}_{11} = \frac{Q_1}{V_1}T_{in} - \frac{Q_1}{V_1}T_{11} - \frac{1}{\rho C_P V_1}q_{11}$$

$$(7)\ \dot{T}_{21} = \frac{Q_1}{V_1}T_{11} - \frac{Q_1}{V_1}T_{21} - \frac{1}{\rho C_P V_1}q_{21}$$

$$(8)\ \dot{T}_{31} = \frac{Q_1}{V_1}T_{21} - \frac{Q_1}{V_1}T_{31} - \frac{1}{\rho C_P V_1}q_{31}$$

$$\vdots$$

$$(9)\ \dot{T}_{251} = \dot{T}_{out1} = \frac{Q_1}{V}T_{241} - \frac{Q_1}{V}T_{251} - \frac{1}{\rho C_P V}q_{251}$$

$$(10)\ \dot{T}_{12} = \frac{Q_2}{V_2}T_{in} - \frac{Q_2}{V_2}T_{12} - \frac{1}{\rho C_P V_2}q_{12}$$

$$(11)\ \dot{T}_{22} = \frac{Q_2}{V_2}T_{12} - \frac{Q_2}{V_2}T_{22} - \frac{1}{\rho C_P V_2}q_{22}$$

$$(12)\ \dot{T}_{32} = \frac{Q_2}{V_2}T_{22} - \frac{Q_2}{V_2}T_{32} - \frac{1}{\rho C_P V_2}q_{32}$$

$$\vdots$$

$$(13)\ \dot{T}_{252} = \dot{T}_{out2} = \frac{Q_2}{V_2}T_{242} - \frac{Q_2}{V_2}T_{252} - \frac{1}{\rho C_P V}q_{252}$$

$$(14)\ T_{out} = \frac{T_{out1} + T_{out2}}{2}$$

The states for the state space notation are presented from (15) until (20). The system inputs are shown in (21).

$$(15)\ x_1 = T_{11}$$

$$(16)\ x_2 = T_{21}$$

$$(17)\ x_{25} = T_{251} = T_{out1}$$

$$(18)\ x_{26} = T_{12}$$

$$(19)\ x_{27} = T_{22}$$

$$(20)\ x_{50} = T_{252} = T_{out2}$$

$$(21)\ u = \begin{vmatrix} T_{in} \\ q_{11} \\ q_{21} \\ \vdots \\ q_{12} \\ q_{22} \\ q_{252} \end{vmatrix}_{51\times 2}$$

From here it follows the state space matrices A, B, C and D, given below.

$$(22)\ A = \begin{vmatrix} -\frac{Q_1}{V_1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \frac{Q_1}{V_1} & -\frac{Q_1}{V_1} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -\frac{Q_2}{V_2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \frac{Q_2}{V_2} & -\frac{Q_2}{V_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \frac{Q_2}{V_2} & -\frac{Q_2}{V_2} \end{vmatrix}_{50\times 50}$$

$$(23)\ B = \begin{vmatrix} \frac{Q_1}{V_{1_{[1,1]}}} & -\frac{1}{\rho C_P V_{1_{[1,2]}}} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -\frac{1}{\rho C_P V_{1_{[2,3]}}} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{Q_2}{V_{2_{[26,1]}}} & 0 & 0 & \cdots & 0 & -\frac{1}{\rho C_P V_{2_{[26,27]}}} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -\frac{1}{\rho C_P V_{2_{[27,28]}}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -\frac{1}{\rho C_P V_{2_{[50,51]}}} \end{vmatrix}_{50\times 51}$$

$$(24)\ C = \begin{vmatrix} 0_{[1,1]} & \cdots & 0_{[1,24]} & 1_{[1,25]} & 0_{[1,26]} & \cdots & 0_{[1,50]} \\ 0_{[2,1]} & \cdots & 0_{[2,24]} & 0_{[2,25]} & 0_{[2,26]} & \cdots & 1_{[2,50]} \end{vmatrix}_{2\times 50}$$

$$(25)\ D = \begin{vmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{vmatrix}_{50\times 51}$$

The model represented by (22)-(25) is the full model of the system, of order 50, which will be reduced using the proposed technique in the following sections.

3. CONTEXT OF LOW-ORDER CONTROLLERS

3.1. Theoretical Background.

Today, a mathematical model (MM) derived to represent the system dynamics for a plant is more complex compared to the same model which could have been obtained several decades ago. The reasons are: i) increasing the performance specifications, ii) increasing demands on productivity, iii) increasing demands on quality, iv) increasing demands on accuracy of modeling, etc. For these MM (often MIMO) which are more complex and therefore more accurate, the obtained controllers are of the same complexity and usually numerically high-order.

However, complex models are not always required in order to achieve good control performance. For example, an unfortunate fact is that optimization methods (i.e. procedures based on H_∞, H_2 H_∞ control, μ - analysis and synthesis, etc) tend to produce controllers with an order at least as many states as the plant model (Choi et. al, 1994; Gu et. al, 2005; Zhou et. al, 1996; Safonov and Chiang, 1989). These high-order controllers are difficult to implement, have a high cost, low numerical reliability, many maintenance problems etc.

Because in control engineering practice a good controller often requires simple and low order functions, the remainder of the paper enumerates the main methods to find less-complex, low-order approximations for plant and controller models (Choi et. al, 1994; Gu et. al, 2005; Zhou et. al, 1996; Safonov and Chiang, 1989; Safonov et. al, 1987; Skogestad and Postlethwaite, 1996; Balas et. al, 1995).

Thus, to obtain a lower-order plant model or a lower-order controller for a high-order plant in literature one can find the following logical ideas: (a) plant model reduction and just after controller design; (b) first step is controller design and the second is controller- order reduction; (c) a direct design of low-order controllers. As in all approaches with many possibilities, each of them has different advantages and drawbacks. However, only the above mentioned (a) and (b) methodologies are useful within the robust control field.

3.2. When should we employ a model-order reduction?

Not all designs of robust controllers for high-order plant or relatively high-order plant (i.e. for all complex systems) require model reduction. However, there are some cases in which this step is mandatory:

(i) – a smaller size model but representative (with dynamics preserved) is desirable for the control designer to *speed up* the simulation process in design stage;

(ii) - a smaller size model but representative is necessary be used *to obtain some specifications*;

(iii) – as above, when the resultant controller using an optimization method (based on H_∞, H_2, μ) is one with an order at least as many states as the plant model, i.e. *greater than needed*, and as a result they are hard to be implemented in practice.

3.3. Classes of Approximation Methods.

Hitherto, there are a manifold of methods available for model-order reduction. From these, for stable systems only, the most used three methods, based on absolute – error approximation, are:

(a)-Balanced Truncation-method which gives a good approximation over high-frequency ranges;

(b)-Singular Perturbation Approximation (or Balanced residualisation) - method which perform better approximation of Bode characteristic over low-frequency and medium frequency ranges; and

(c)-Hankel-Norm Approximation - method which usually perform better approximation at high frequency.

When a system $G(s)$ is *unstable*, first one can apply modal decomposition to find a stable $G_{st}(s)$ and an unstable part $G_{unst}(s)$ (all poles in RHCP), $G(s) = G_{st}(s) + G_{unst}(s)$. It is then possible for $G(s)$ to be reduced to $G_{stRED}(s)$ with any from above methods. Another method applicable to the *unstable* systems is the one obtained through reduction of normalized coprime factors of the system.

When a reduced-order model is necessary for a practical application to approximate equally well the Bode magnitude over the whole frequency range, the method used is a stochastic one, namely *balanced stochastic truncation* (BST) (Desai and Pal, 1984; Green, 1988).

Finally, in those cases when it is necessary not only to obtain a particular reduced-order controller, but also some design specifications of the closed-loop system, the controller-order reduction problem can be better formulated in a frequency framework, i.e. as a frequency-weighted model reduction (Gu et. al, 2005).

3.4. Practical model reduction methods.

Two categories from above methods are to minimize the H_∞ norm between the full order model and the reduced order model, $\|G(s)-G_{RED}(s)\|_\infty$.

The *first* algorithm for model approximation and order reduction, controls the *absolute approximation error*, and is based on the Hankel singular values of the system. In this case (the additive error), the reduction algorithm returns a reduced order model G_{RED} of the original model G with a bound on the error, the peak gain across frequency. In (Glover, 1984) is shown that the reduced order model $G_{RED}(s)$ of the original model $G(s)$ has a bound on the infinity norm of the error, $\|G(s)-G_{RED}(s)\|_\infty$, which must satisfy the inequality

$$(26)\quad \|G(s)-G_{RED}(s)\|_\infty \le 2\sum_{k+1}^{n}\sigma_i$$

where σ_i is i^{th} Hankel singular value of the original system G.

The *second* algorithm for model approximation and order reduction, controls the *multiplicative (or relative) approximation error*, and is based on the Hankel singular values of the system. In the latter case (the multiplicative (relative) error), the reduction algorithm returns a reduced order model G_{RED} of the original model G with a bound on the relative error $\|G^{-1}(s)(G(s)-G_{RED}(s))\|_\infty$, which must satisfy the inequality, (Zhou et. al, 1996):

$$(27)\quad \|G(s)-G_{RED}(s)\|_\infty \le \prod_{k+1}^{n}\left(1+2\sigma_i\left(\sqrt{1+\sigma_i^2}+\sigma_i\right)\right)-1$$

If in control theory, the eigenvalues (λ_i) show the system *stability*, in robust control the Hankel singular values (σ_H) show the "*energy*" of each state in the system. The idea is to keep (only) larger "energetic" states of a system, i.e. states which preserves most of the system characteristics (as stability, frequency and time responses etc).

Most model reduction techniques from software packages such as MATLAB or Slicot, used in practical applications, are based on the Hankel singular values (HSVs) of a system. The HSVs can obtain a reduced order model that preserves the majority of the system dynamic characteristics (see above idea). For a stable state-space system (**A**, **B, C, D**), its HSVs are defined, (Glover, 1984), as

$$(28)\quad \sigma_H = \left(\lambda_i\left(\mathbf{PQ}\right)\right)^{1/2}$$

where **P** and **Q** are *Controllability* and *Observability Grammians* satisfying following Lyapunov equations:

$$(29)\quad \mathbf{AP}+\mathbf{PA}^T = -\mathbf{BB}^T$$

$$(30)\quad \mathbf{A}^T\mathbf{Q}+\mathbf{QA} = -\mathbf{C}^T\mathbf{C}$$

As a conclusion, for practical applications, the most used model reduction methods are: (a) *absolute-error approximation* (or *additive error method*), and (b) *relative-error approximation* (or *multiplicative error method*). In both of the above categories, the reduced order model has: in (a) - an additive error bounded by an error criterion and in (b) - a multiplicative (or relative error) bounded by an error criterion. In both methods, the error is measured in terms of peak gain across frequency, i.e. H_∞ norm, and the error bounds are a function of the neglected Hankel singular values, $\|G(s)-G_{RED}(s)\|_\infty$. In other words: performance preservation indicates that the H_∞ norm bound of the closed loop transfer function with *reduced-order* controller is not greater than the H_∞ norm bound of the closed loop transfer function with *full order* controller. In any case from the two above, is assumed *additive*, respectively multiplicative perturbation to the closed loop transfer function, to obtain sufficient conditions for performance preservation.

4. RESULTS AND DISCUSSIONS

In this section we apply the method described in section 3 on the model from section 2. Figure 2 depicts the Bode characteristic of the full system from figure 1.

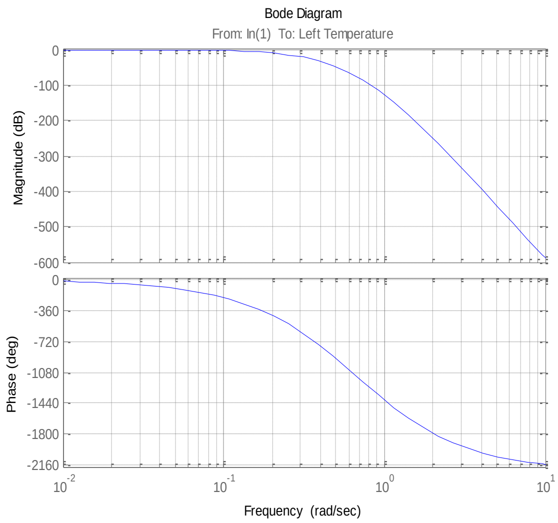

Fig.2. Bode Plot of the full-order system from figure 1, with 25 states in one branch.

The application of the Hankel matrix decomposition delivers the information that of the total of 50 states, only 16 are dominant, as shown in figure 3 below. From figure 3, one may observe that the number of states with high energy in the Hankel singular value matrix can be reduced to 6, while preserving the original dominant dynamic characteristics of the original system from section 2. Figure 4 depicts these 6 dominant states.

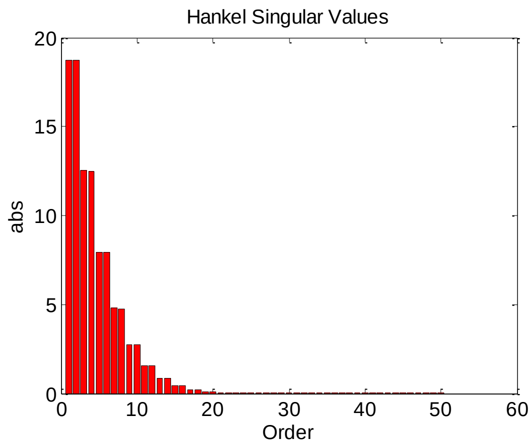

Fig.3. The result of the Hankel matrix decomposition in singular values, suggesting a lower order model from the full one with 50 states.

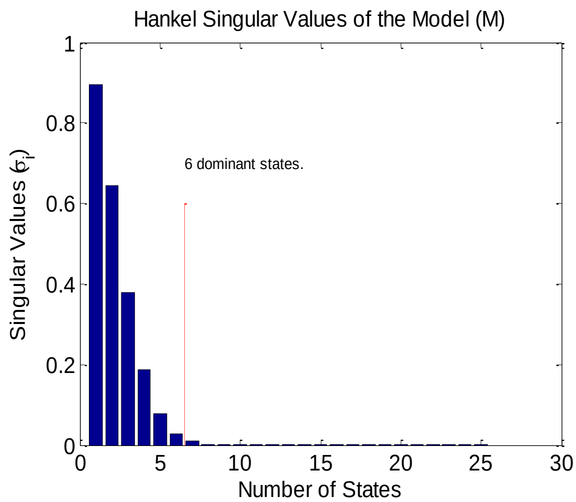

Fig.4. Selection of the 6 dominant states minimally necessary to preserve the dominant dynamics of the original system over one branch of figure 1 computed through the first method (additive error method).

Method 1: In order to realize the reduced model, we use the absolute error method (also known as additive error method). We therefore apply a balanced truncation algorithm, by means of the Matlab(R) function **balancmr**. This method has the advantage that the error introduced by the remaining states (25-6=19) is uniformly minimized in the frequency interval of interest. From the Bode plot (not shown) it can be observed that the method works well in the low frequency range. However, above 0.5rad/s, the reduced order model does not capture well the original dynamics of the system.

Method 2: We also employ a second method, namely that of relative error (also known as multiplicative error method). Such algorithm employs a balanced stochastic truncation (Schur method) and is very effective for all kinds of processes (linear, nonlinear, continuous or discrete). The Matlab(R) function which applies this method is **bstmr**. The algorithm computes the infinity-norm of $\left\| M^{-1}\left(M - Mr\right)\right\|_{\infty}$, hence it may cause numerical issues if the gain is close to 0dB (not in our case). Figure 5 depicts the selection of the 8 dominant states and from the Bode plot (not shown) we also conclude that the low frequency range is well approximated.

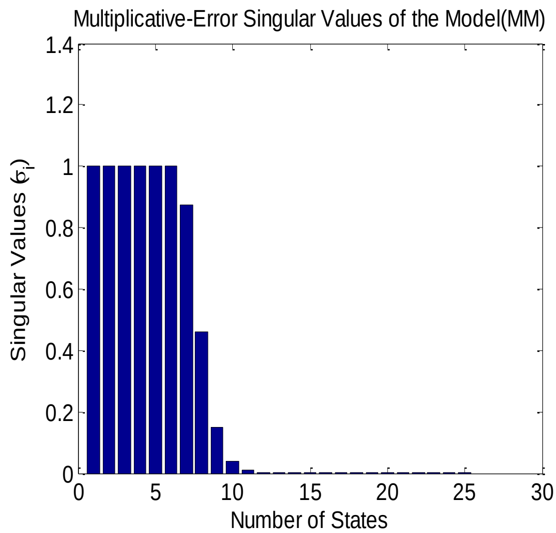

Fig.5. Selection of the 8 dominant states minimally necessary to preserve the dominant dynamics of the original system over one branch of figure 1 computed through the second method (multiplicative error method).

However, when results from both methods are compared in terms of time-domain (i.e. step response), they deliver unstable results. Therefore, we apply one more step and select 7 states with a balanced truncation method for both additive and multiplicative error methods. The Bode plot result is given in figure 6. The step response is given in figure 7 for the methods employed, where is shown clearly that the additive error method outperforms the multiplicative error method.

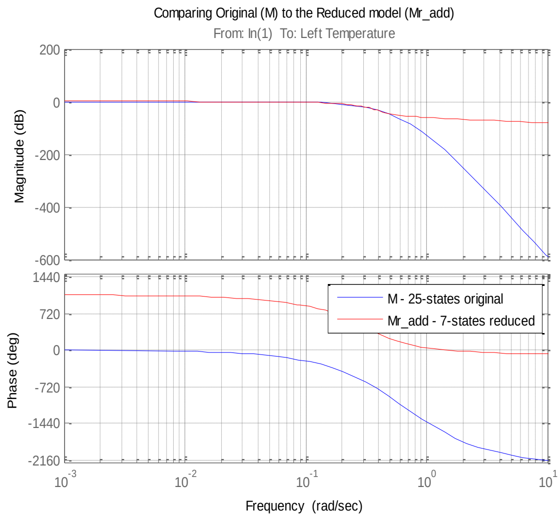

Fig.6. Bode plot of the reduced order model with 7 states.

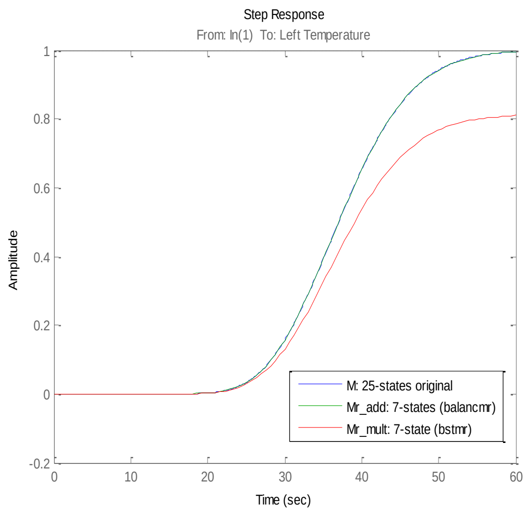

Fig.7. Step response of the original method and the reduced methods. Notice that the reduced model with additive error method outperforms the multiplicative error method result.

5. CONCLUSIONS

To conclude, we have presented in this paper an overview of simple yet effective methods for model order reduction. The next objective is to test these methods on a nonlinear system (through linearization techniques) and design robust controllers.

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