THE ANNALS OF "DUNAREA DE JOS" UNIVERSITY OF GALATI
FASCICLE III, Vol.31, No.2, ISSN 1221-454X
ELECTROTECHNICS, ELECTRONICS, AUTOMATIC CONTROL, INFORMATICS

PID SELF-TUNING CONTROLLERS USED IN DIFFERENT STRUCTURES OF THE CONTROL LOOPS

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Abstract: The paper investigates the behavior of digital PID self-tuning controllers (STC) in different structures of the control loops used in adaptive systems. In the two phases of this type of systems which use a STC-PID, the first phase, i.e. the task of recursive identification of the plant model parameters, is used a regression (ARX) model with the recursive least squares method. Because the quality of process model depends on the order of the ARX model and of the sample period (T_s), the digital PID parameters are functions of these variables and, supplementary, of continuous-time PID parameters and of the control loop structures used in the adaptive system (although, not so largely as the firstly three variables). To see the latter influence, in this paper are considered two control loop block diagram, and for simulations - three different processes (stable; with no minimum phase; unstable), and some simulations with different T_s, ω_n, ξ . The PID controller design method used to obtain the specs desired for control loop dynamic behavior was the pole assignment method of the loop.

Keywords: STC- PID adaptive systems, different feedback loop structures, simulation, specs comparation.

1. INTRODUCTION

In recent years, the theory of adaptive control has made more and significant developments. After [1], the basic approaches to the problem of adaptive control are *gain-scheduling* (GS), *model-reference adaptive control* (MRAC), *self-tuning controllers* (STC) and *dual control* (DC).

If the estimates of the process parameters are adjusted and the controller parameters are obtained from the solution of a design problem using the estimated parameters, the system is viewed as an automation of process modeling and design. In this case the process model and control design are updated at each sampling period. This controller is called *self-tuning controller* (STC) or *self-tuning regulator* (STR) because it identifies unknown processes firstly, and then synthesizes the control (i.e. adaptive control with recursive identification).

Hitherto, the most useful "STC results" have been achieved, mainly, in SISO systems, for which were designed some stable algorithms with different complexity.

Depending on the nature of the controlled process and on the general task of optimal adaptive control with recursive identification, the STCs can be *implicit* or *explicit*. In the STCs where the identification process does not serve to determine

This paper was recommended for publication by Adrian Filipescu

estimates of the process model parameters, but are used recursively to estimate the controller parameters *directly*, they are referred as being *implicit* (Fig. 2). Consequently, when the STCs use a synthesis from estimates of the process model parameters (*indirect* identification), these are called *explicit.* (Fig. 1).

In any adaptive system, both phases representing recursive identification and controller parameter calculation are valid only at the moment when the STC is being set up, i.e. during the adjustment phase. After the STC has been adjusted, these are “non-valid” because the identification is switched off and the system is controlled with fixed parameters.

In adaptive control system both task of identification and control synthesis have with the same level of importance.

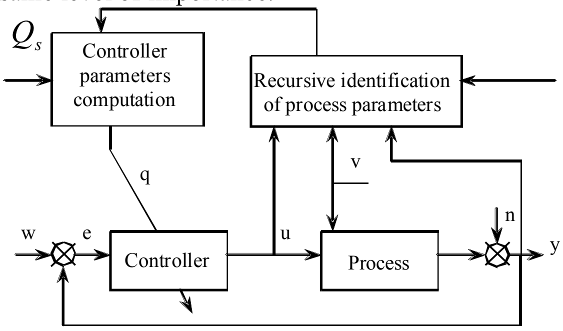

Fig. 1: Block diagram of an explicit STC (with direct identification)

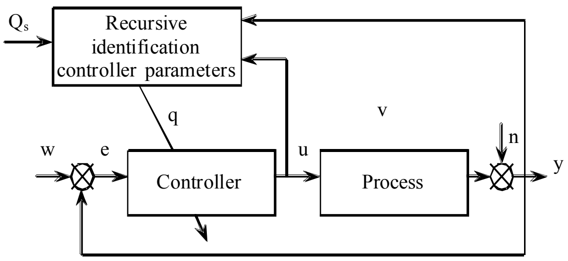

Fig. 2: Block diagram of an implicit STC (with indirect identification)

As a result, in the majority of practical cases where STCs are designed and used to estimate their parameters is used an ARX regression model. Much more, the least square method described in [13, 8], has given the best result in on-line estimation of the ARX parameters [10, 3, 1, 6].

In the following, the paper has the intention to asses the difference between the behavior of a STC derived with a *standard block diagram* of a closed loop (Fig.3, with 1DOF) and the behavior obtained with a different (better) block diagram (see Fig. 4, with 2DOF). In the above both cases, the STCs and closed feedback loops are based on the poles assignment of the loop. Using suitable pole configurations, it is possible to fulfill the specs for stability and a desired closed loop-response (as overshoot, damping factor, rise-time etc.).

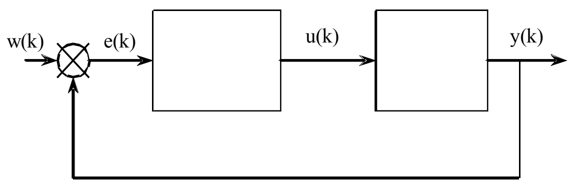

Fig. 3: The first block diagram of a *standard* control loop with STC-PID controller, named with a one degree of freedom (1DOF)

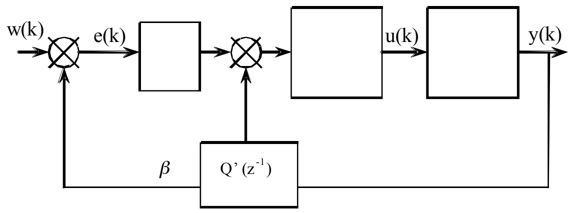

Fig. 4: The second block diagram of a control loop with STC-PID controller, derived from the structure with two degree of freedom (2DOF) [12]

2. STANDARD CONTROL LOOP WITH STC PID DIGITAL CONTROLLERS

Usually, the continuous-time PID controller equation from Fig. 3 or Fig. 4 is, [12, 2, and 3]:

$$u(t) = K_P\left[e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D\frac{de(t)}{dt}\right] \qquad (1)$$

u(t) being the controller output, y(t) - process output (controlled/ manipulated variable), e(t)- tracking error and w(t)- the reference signal (set point); K_P, T_I and T_D are the controller parameters, respectively proportional gain, integral time constant and derivative time constant.

Using the Laplace transformation in (1), we obtain the transfer function of this controller:

$$H_C(s) = \frac{U(s)}{E(s)} = K_P\left[1 + \frac{1}{T_I s} + T_D s\right] \qquad (2)$$

As in [9, 5, 4, and 6], using a (small) sampling period to discretize the integral and derivative components of (1), can be obtained the two digital forms of the PID algorithm:

i) the position algorithms of the PID controllers (such named because it final output is the *manipulation*, also know as the *control actuator position)*:

$$u(k)=K_P\left\{\begin{array}{l} e(k)+\dfrac{T_s}{T_I}\left[\dfrac{e(0)+e(k)}{2}+\sum_{i=1}^{k-1}e(i)\right]+ \\ +\dfrac{T_D}{T_s}\left[e(k)-e(k-1)\right] \end{array}\right\} \quad (3)$$

where T_s is the sampling time (period) of discretising the integral and derivative of continuous-time error e(t), at points e (kT_s), k = 0,1,2,3,….

To discretize the derivative component of (3), the most algorithms use a difference of the first order (two-point, backward difference) and to approximate the integral (by simple summing) are used three methods: (a) *forward rectangular*; (b) *backward rectangular* and (c) *trapezoidal* one (more accurate). However, all three algorithms are called *nonrecurrent algorithms,* because *all* previous error values e (k-1), i = 1, 2, k, have to be known to calculate the integral, and after the controller action.

ii) *velocity algorithms* or *incremental algorithms* for the PID controllers are algorithms which calculate the increment (the change) Δu (k). They can be determined from (3) by obtaining u (k-1) from u (k), and substracting the resulting expressions. The recurrent relation u(k) = Δu(k) + u(k-1) used in (3) is

$$\Delta u(k)=K_P\left\{e(k)-e(k-1)+\frac{T_s}{T_I}e(k)+\right. \\ \left.+\frac{T_D}{T_s}\left[e(k)-2e(k-1)+e(k-2)\right]\right\} \quad (4)$$

and, in general form:

$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1) \quad (5)$$

Depending on the three methods used to approximate the continuous-time function by sampling periods T_s of the constant function (step, rectangle, i.e. forward rectangle, backward rectangular or trapezoidal method), can be obtained three different incremental controller parameters q_o, q_1, q_2 , as functions of K_P, T_I, T_D and T_s parameters:

$$q_k= f (K_p, T_I, T_D, T_s),\ \ k = 0, 1, 2. \quad (6)$$

For example, in the recurrent relation (5) obtained from (4), the controller parameters are:

$$\begin{aligned} &q_0 = K_p (1+T_s /T_I + T_D /T_s),\\ &q_1 = - K_p (1+2T_D/Ts),\\ &q_2=K_pT_D/Ts \end{aligned} \quad (7)$$

Much more, these parameters are functions not only as in (6), but depend of the discretization method, and of the closed loop block diagram.

The characteristic polynomial of the standard control loop with a STC

The discrete transfer functions of the controlled process and PID controller are (see Fig. 3, z^{-1} = backward time-shift operator, i.e. x (k-1) = z^{-1}x (k)):

$$H_P(z)=\frac{Y(z)}{U(z)}=\frac{B(z^{-1})}{A(z^{-1})}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \quad (8)$$

and

$$H_R(z)=\frac{U(z)}{E(z)}=\frac{Q(z^{-1})}{P(z^{-1})}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{(1-z^{-1})(1+\gamma z^{-1})} \quad (9)$$

In (9), the PID controller discrete transfer function *standard form*, i.e.

$$H_R(z)=\frac{Q(z^{-1})}{P(z^{-1})}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{(1-z^{-1})},$$

is used together with a serially connected digital filter:

$$H_F(s)=\frac{1}{1+\gamma z^{-1}} \quad (10)$$

filter used to compensate unwanted interference of the expression $(1+\gamma z^{-1})$ $(\gamma = b_1/b_0)$.

From (9) can be obtained the control (command) action

$$U(z)=\frac{Q(z^{-1})}{P(z^{-1})}E(z), \quad (11)$$

Inserting here the polynomials of Q (z^{-1}) and P(z^{-1}), the controller output using difference equations is

$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+ \\ +(1-\gamma)u(k-1)+\gamma u(k-2) \quad (12)$$

From the standard block diagram shown in Fig. 3, the closed loop transfer function is

$$H_0(z)=\frac{Y(z)}{W(z)}=\frac{B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})} \quad (13)$$

To fix the *desired* pole assignment for above closed loop transfer function is necessary to choose the characteristic polynomial as

$$D(z^{-1})=1+\sum_{i=1}^{n_d}d_iz^{-i},\quad n_d\le 4\ (q_0,q_1,q_2,\gamma) \quad (14)$$

for the denominator of (13), i. e. for

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \qquad (15)$$

In other words, to achieve the specs is necessary to select the correct parameters for controller polynomials (11), which are the solutions of polynomial (15).

We know from [6], that the most frequently used method of pole assignment to obtain a required control response of a closed loop, is done by selecting natural frequency ω_n and damping factor ξ in the characteristic equation for a second order plant, as the polynomial D(s) = $s^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$.

For the polynomial form of D (z^{-1}) have been chosen

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} \qquad (16)$$

For a sampling period T_s, the coefficients of (16) are, in this case:

$d_1 = -2\exp(-\xi\omega_n T_0)\cos(\omega_n T_0\sqrt{1-\xi^2})$, if $\xi \le 1$;

$d_1 = -2\exp(-\xi\omega_n T_0)\cosh(\omega_n T_0\sqrt{1-\xi^2})$, if $\xi > 1$;

$d_2 = \exp(-2\xi\omega_n T_0)$.

Equating (16) in the right side of (15), the four unknown controller parameters can be obtained [9], as functions of the controlled system parameters and of the number of poles and their desired position in the *z* complex plane.

3. THE SECOND STRUCTURE OF THE CONTROL LOOP WITH A STC PID CONTROLLER

Comparing the first standard loop with the second closed-loop block diagram from Fig. 4 designed in [3] and [4], the polynomial P (z^{-1}) has the same form as polynomial P (z^{-1}) of the PID controller denominator discrete transfer function, i.e.

$$P(z^{-1}) = (1 - z^{-1})(1 + \gamma z^{-1}) \qquad (17)$$

In Fig. 3, the controller transfer function is

$$H_R(z) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{(1 - z^{-1})} \qquad (18)$$

and the discrete transfer function of the process is

$$H_P(z) = \frac{B(z^{-1})}{A(z^{-1})} = \frac{[(b_0 + b_1 z^{-d})]}{(1 + a_1 z^{-1} + a_2 z^{-2})} \qquad (19)$$

where $b_0 \neq 0$ and d > 0 is the number of the time delay steps. In the controller equation from the second closed loop structure in Fig. 4, i.e. in

$$U(z) = [\beta E(z) - Q'(z^{-1})Y(z)]\frac{1}{P(z^{-1})} \qquad (20)$$

the polynomial P (z^{-1}) has the same form as above polynomial (18) for the first controller (Fig. 3). The polynomial Q (z^{-1}) from (18), is different here and has the form (21)

$$Q'(z^{-1}) = (1 - z^{-1})(q_0' - q_2' z^{-1}) \qquad (21)$$

Using P (z^{-1}) from (17) and Q' (z^{-1}) from (21) in (20), can be obtained controller output as:

$$\begin{aligned} u(k) = &-[(q_0' + \beta)y(k) - (q_0' + q_2')y(k-1) + \\ &+ q_2' y(k-2)] - (\gamma - 1)u(k-1) + \\ &+ \gamma u(k-2) + \beta w(k) \end{aligned} \qquad (22)$$

The closed loop transfer function from Fig. 4 is

$$\begin{aligned} H_0(z) &= \frac{Y(z)}{W(z)} = \\ &= \frac{\beta B(z^{-1})}{\{A(z^{-1})P(z^{-1}) + B(z^{-1})[Q'(z^{-1}) + \beta]\}} \end{aligned} \qquad (23)$$

where the characteristic polynomial equation (i.e. the denominator) is:

$$D(z^{-1}) = A(z^{-1})P(z^{-1}) + B(z^{-1})[Q'(z^{-1}) + \beta] \qquad (24)$$

If the controlled process discrete transfer function has the same polynomials A (z^{-1}) and B (z^{-1}) as in (19), using (24) can be found the four unknown controller parameters q_0', q_2', β and γ (q_1'= 0 in the polynomial Q' (z^{-1})).

As above in Fig. 3, in Fig. 4 was used the same pole assignment method, by choosing the characteristic polynomial of the form (14) in the polynomial (24), to assign the desired pole placement for the closed loop transfer function (13).

4. THE EFFECTS OF THE DIFFERENT DISCRETE STC-PID PARAMETERS

In order to observe the effects of the different discrete STC-PID parameters obtained with the two different control loop block diagram, were considered three kind controlled process (c.p.) with the following transfer functions:

i) A stable c.p.: $H_1(s) = 1/[(3s+1)(s+1)]$;

ii) A nonminimum phase c.p., H_2:

$H_2(s) = (-s + 1)/ [(4s+1)(s+1)]$;

iii) An unstable c.p., $H_3(s) = (s + 1)/ [(s+1)(4s-1)]$;

With $T_s = 1$, the discretized transfer functions are:

$$H_1(z) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} = \frac{0.1091z^{-1} + 0.07004z^{-2}}{1 - 1.084z^{-1} + 0.2636z^{-2}}$$

$$H_2(z) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} = \frac{-0.05275z^{-1} + 0.1927z^{-2}}{1 - 1.147z^{-1} + 0.2865z^{-2}}$$

$$H_3(z) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} = \frac{0.284z^{-1} - 0.1045z^{-2}}{1 - 1.652z^{-1} + 0.4724z^{-2}}$$

Using a damping factor $\zeta=1$ (unperiodic critic), sampling-time $T_s=1$s and different values for natural frequency (to observe the recommended values from [1], i.e. $0.45 \le \omega_n T_s \le 0.9$ were obtained different STC-PID parameters and the particular recurrent equations (12), and (22) respectively, for each case.

The results obtained for dynamic behaviors of the three processes (systems) are shown below. For each STC-PID controller designed and used in the *standard control loop* Fig. 3 with the three above processes, were simulated and shown: step response for $H_1(s)$, and Simulink results (*y, w* and *u* variables of the $H_1(z)$ model), in Fig. 5, 6, 7); the same simulations for the $H_2(s)$ and $H_2(z)$ models in Fig. 8, 9, 10); finally, for the $H_3(s)$ and $H_3(z)$ models, in Fig. 11, 12 and 13, respectively.

The results obtained with the second closed loop block diagram for the same three models in the same fashion, are shown in the Fig. 14, 15, 16 ($H_1(s)$, $H_1(z)$), the Fig. 17, 18, 19 ($H_2(s)$, $H_2(z)$), and the Fig. 20, 21, 22 ($H_3(s)$, $H_3(z)$), respectively.

Results: the first control loop structure (Fig. 5-13, without Fig. 12, i.e. the y, w variables of H_3 (z)):

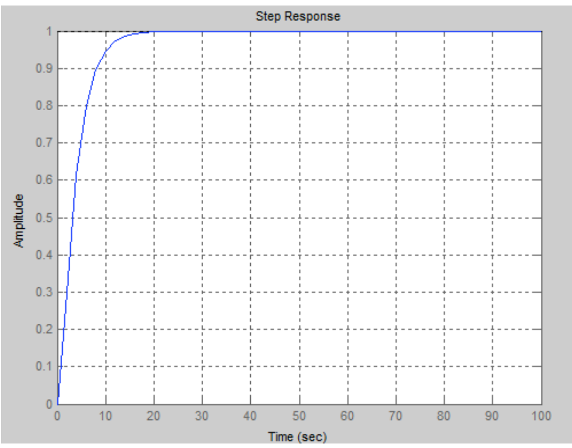

Fig. 5: Step response, $H_1(s)$

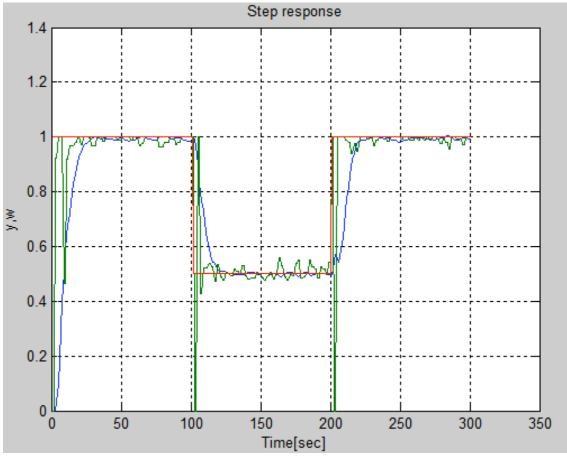

Fig. 6: The variables y, w, H_1 (z)

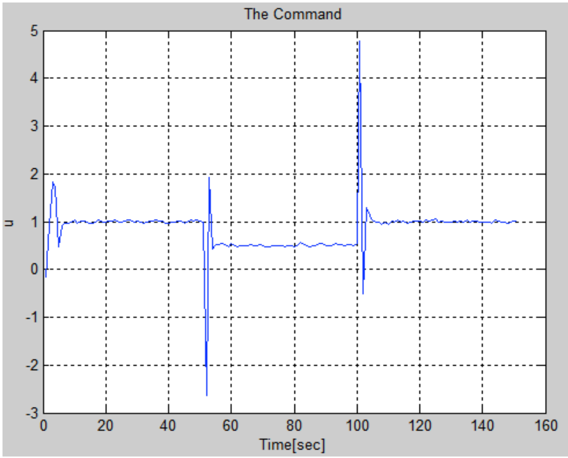

Fig. 7: The command *u*, H_1 (z)

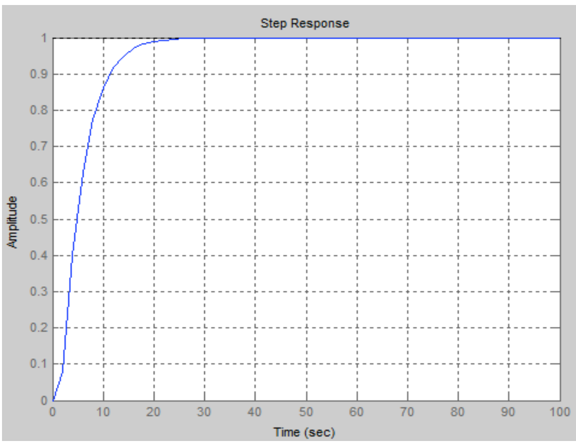

Fig. 8: Step response, H2 (s)

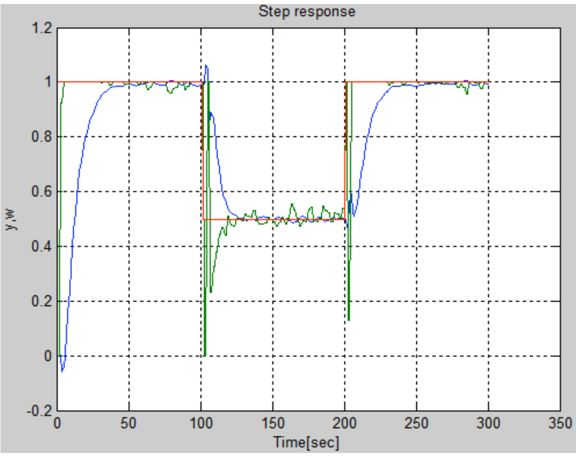

Fig. 9: The variables y, w, H_2 (z)

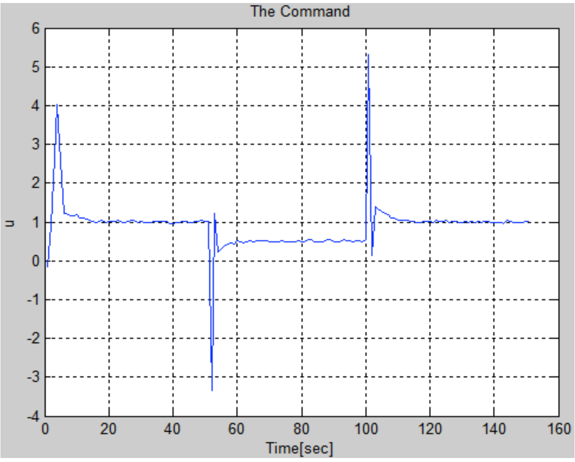

Fig.10: The command *u*, H_2 (z)

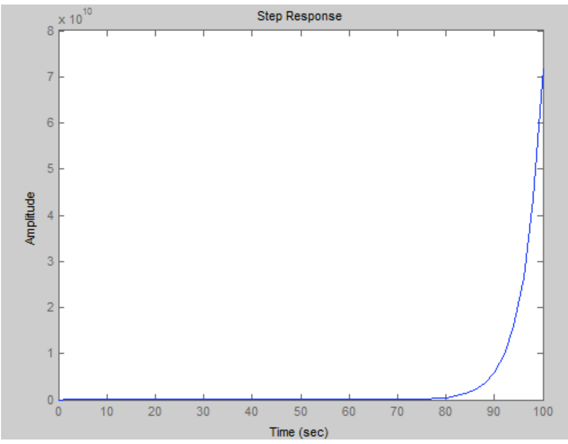

Fig. 11: Step response, H3(s), the unstable system

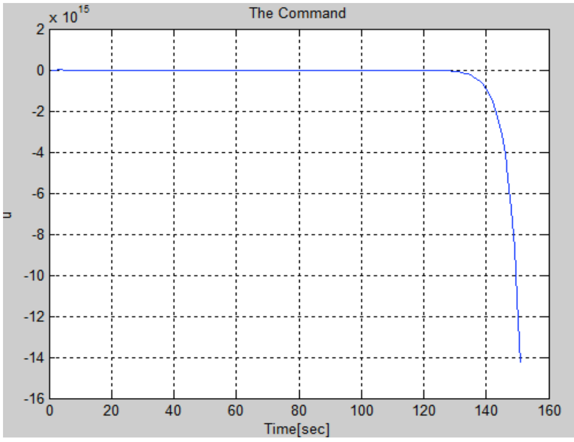

Fig. 13: The command *u*, H_3 (z), the unstable system

Simulations obtained with the second loop structure (Fig. 14-22, without Fig. 21, i.e. the y, w variables of H_3 (z)):

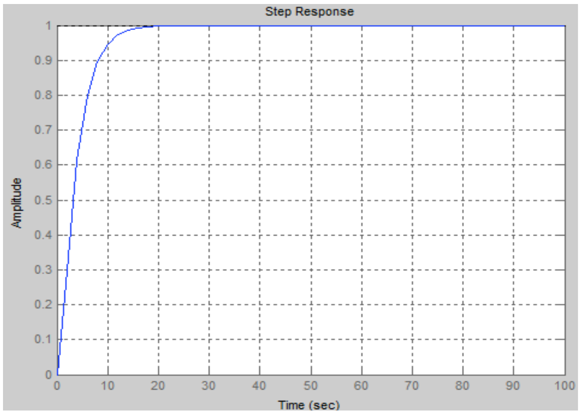

Fig. 14: *Step response, H_1(s)

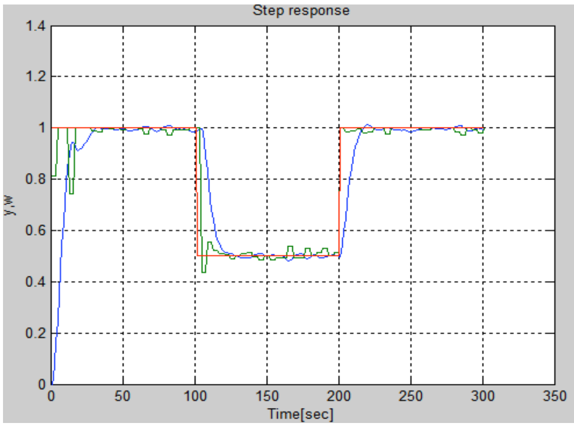

Fig.15:*The variables y, w, H_1 (z)

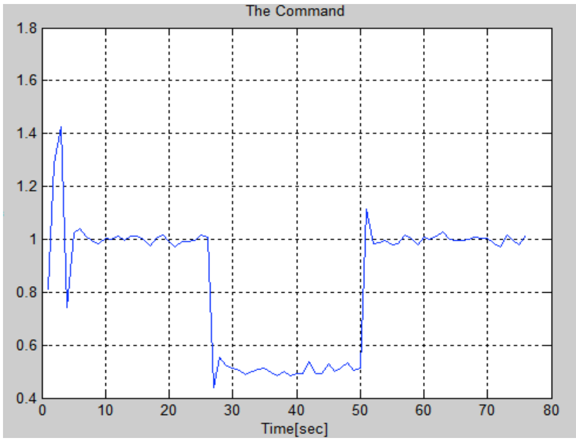

Fig.16: *The command *u*, H_1 (z)

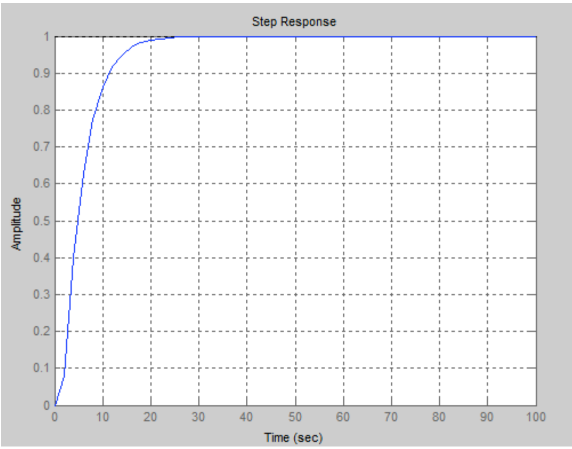

Fig. 17: *Step response, H_2(s)

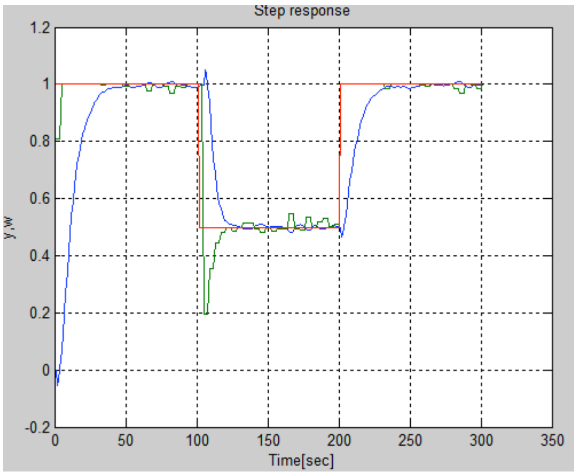

Fig. 18:*The variables y, w, H_2 (z)

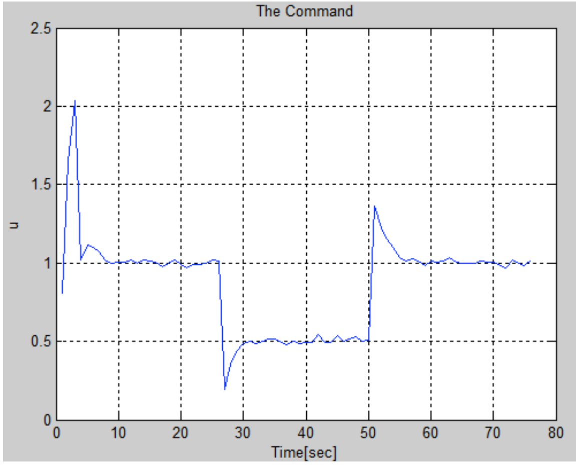

Fig. 19:*The command *u*, H_2 (z)

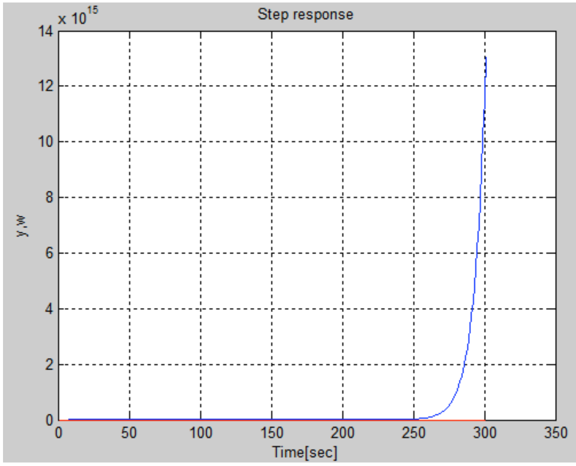

Fig. 20:*Step response, H_3(s), the unstable system

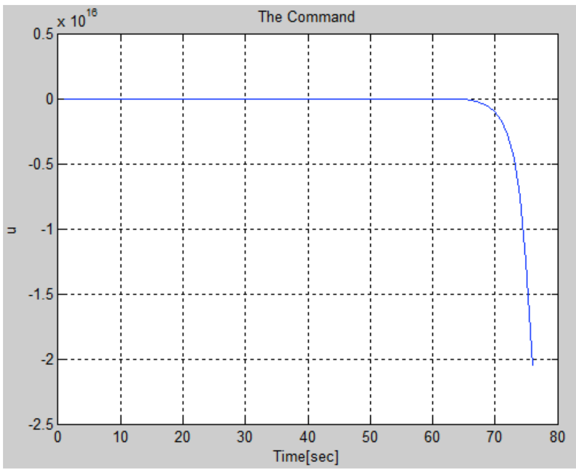

Fig. 22:*The command *u*, H_3 (z), the unstable system

5. CONCLUSIONS

From the above design and simulation results, the following conclusions can be drawn:

1. The quality of control with digital STC-PID is affected by some parameters as e.g. sampling period, continuous-time PID parameters, control law algorithm, actuator saturation, initial parameter estimation, recursive least squares (RLS) method used to on-line identification, the type of closed-loop control block diagram (1DOF or 2DOF).
2. The two closed-loop control block diagram and the simulations for any of the three processes used in the paper ($H_i(s)$, I= 1, 2, 3), have, each, a different controller equation, i.e. (12) or (22), with different controller parameters.
3. For design parameters of STC-PID was used pole placement method via characteristic polynomial (16), i.e. $D(z^{-1}) = 1+d_1z^{-1}+d_2z^{-2}$, real roots, to obtain a similar dynamic behavior to that of second-order continuous-time systems with a characteristic polynomial $D(s) = s^2 + 2\xi\omega_n s + \omega_n^2$ (where the dominant poles are given by desired damping factor ξ and the natural frequency ω_n of the closed loop). In all designs the damping factor was ξ =1 but different ω_n, being respected the inequality $0.45 \leq \omega_n T_s \leq 0.9$ from the reference [1].
4. The responses were compared to see the conventional specs: percent overshoot (POS); rise-time (t_r); settling-time (t_s) and steady-state error (ε_{ss}).
5. As a final conclusion resulted from all responses, the second closed-loop block-diagram (2DOF) when is used with STC-PID, give better results as in the first case, standard case (1DOF).

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