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DEVELOPMENT OF A SIMULINK® TOOLBOX FOR FRICTION CONTROL DESIGN AND COMPENSATION

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Abstract: This paper focuses on the development of a MATLAB/Simulink® library for servo-systems with friction as a part of a new simulation platform dedicated to model, analysis and control design of friction. It is well known that friction is a very important process for the control engineering both for high-precision servo – mechanisms and simple pneumatic and hydraulic systems. Highly nonlinear process, friction may result in steady state errors, limit cycles and poor performance. It is therefore important for control engineering to understand friction phenomena and to know how to deal with them. Moreover, a reliable library of friction models that captures the friction behavior provides an important tool in order to investigate by analysis and simulation the properties of friction that are relevant to control design.

Keywords: Stiction, Stribeck effect, KFM, pre-sliding displacement, stick-slip motion, Model-based friction compensation;

1. INTRODUCTION

Friction is one of the greatest obstacles in high precision positioning systems. It can cause steady state and tracking errors, while it may result in limit cycles. Therefore, its influence on the response of systems, such as a servomechanism, must be seriously considered. Control strategies that attempt to compensate for the effects of friction inherently require a suitable friction model to predict and to compensate for the friction. A good friction model is also necessary to analyze stability, predict limit cycles, find controller gains and perform simulations. Subsequently, the purpose of this paper is to collect a number of friction models developed by author in MATLAB/Simulink® software environment and make them available as a tool in the process of control strategy design for servo-systems with friction. Model-based compensation techniques and several simulation tests using the library models are also presented in order to validate the experimental results already known in literature.

The development of a library containing the main friction models as a Simulink® toolbox was headed to get a good and readily available friction model in order to simulate process applications involving friction and all the issues emerged from it. Hereby, the concept of the library as a flexible simulation tool took into account the overall model requirements:

- openness;
- ease of parameters determination;
- emphasis on simulation speed;
- literature model coverage (further improvement may be added);
- user friendly: documented, reliable;
- easy to extend.

Moreover, beside the typical and latest used friction models considered by author, the library includes several control design components employed in friction compensation and control techniques: PID controller and a friction observer.

The paper is organized as follows. Section 2 presents a number of friction models. All models attempt to capture the essence of the complicated friction phenomena within approaches of reasonable complexity. The natures of models is quite different. They can be static or dynamic. Differential equations or hybrid models that include events can describe

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them. Static and dynamic models are surveyed in 2 paragraphs of section 2. A brief description of library content is given in section 3. The block diagram and some user-friendly features developed in Simulink® for each library model are presented in section 4. To further illustrate the properties of library models some tests that illustrate the friction behavior and its servo problems occurring in control design, were performed and their simulation results are presented in section 5. Some concluding remarks end this paper.

2. FRICTION PHENOMENA AND MODELS

Friction is a natural phenomenon that is quite hard to model, and it is not completely understood. Static maps between velocity and friction force describe the classical friction models used. Typical examples are different combinations of Coulomb friction, viscous friction, and Stribeck effect [B. Armstrong-Helouvry, *et al.*, 1994]. The latter is recognized to produce a destabilizing effect at very low velocities. The classical models explain neither hysteretic behavior when studying friction for nonstationary velocities (so-called Dahl effect) nor variations in the breakaway force with the experimental condition nor small displacements that occur at the contact interface during stiction (short for static friction). Therefore, friction model must include dynamics for a better accuracy in describing friction phenomena. It is important to underline that the more important purpose pursued in conceiving or adopting a friction model is to capture the friction phenomena for low velocities and especially for crossing zero velocity regime.

2.1. Static models

Friction static models have evolved from the simplest relay approach of Coulomb friction, F_C, to a generalized mathematical description which includes viscous term, $F_V v$, stiction, F_S (friction at rest) and Stribeck function. These classical friction components can be combined in different ways and any such combination is referred to as a classical KFM (Kinetic Friction Model). These models, presented in Figure 1, have components that are either linear in velocity or constant (Fig.1.a, b). Stribeck observed [Stribeck, R., 1902] that the friction force does not decrease discontinuously (Fig.1.c) but the velocity dependence is continuous (Fig.1.d). This is called Stribeck friction. A more general description of friction than the classical models is, therefore,

$$(1)\quad F = \begin{cases} F(v) & \mathit{if}\ v \neq 0 \\ F_e & \mathit{if}\ v = 0\ \mathit{and}\ |F_e| < F_S \\ F_S \operatorname{sgn}(F_e) & \mathit{otherwise}\big(v = 0\ \mathit{and}\ |F_e| > F_S\big) \end{cases}$$

where F_e is the external force and $F(v)$ is an arbitrary function. A number of parameterizations of $F(v)$ have been proposed [B. Armstrong-Helouvry and C. Canudas de Wit, 1996] and the library models took them into account. A common form of the nonlinearity is

$$(2)\quad F(v) = F_C + (F_S - F_C)e^{-|v|/v_s} + F_v v$$

where v_s is called the Stribeck velocity.

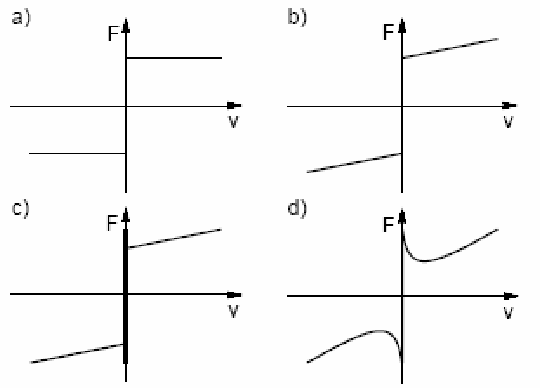


Fig.1. Static friction models: a) Coulomb friction; b) Coulomb plus viscous friction; c) Stiction plus Coulomb and viscous friction; d) example of a parameterization of $F(v)$.

The main disadvantage when using a model such as the one described by equations 1 and 2, for simulation and control purposes, is detecting the zero velocity. A remedy for this is found in the model presented by Karnopp, [Karnopp, D., 1985]. The model defines a zero velocity interval, $|v| < DV$ (hereafter DV denotes this specific interval). For velocities within this interval the internal state of the system (the velocity) may change and be non-zero but the output of the block is maintained at zero by a dead-zone. Depending on if $|v| < DV$ or not, the friction force is either a saturated version of external force or an arbitrary static function of velocity. The author has developed within enclosed static models of library a zero velocity interval variant for each model configuration (see next section). The drawback with this static model approach is that it is strongly coupled with the rest of the considered system – the external force is an input to the model and this force is not always explicitly given. However, this kind of models allows a good representation of stick-slip motion behavior and efficient simulations.

2.2. Dynamic models

The friction models presented so far have considered friction only for steady velocities. No attention is paid to the behavior of friction as the velocity is varied. One of the first dynamical models for friction was proposed by Dahl [Dahl, P. R., 1968]. Describing the spring-like behavior of friction during stiction, the model is essentially Coulomb friction with a lag in the change of friction force when the direction of motion is changed. Thus, the friction force is only a function of the displacement and the sign of velocity. This implies that the friction force is only position dependent and yields a generalization of ordinary Coulomb friction. Unfortunately, the Dahl model neither capture the Stribeck effect, which is a rate dependent phenomenon, nor does it capture stiction. These are the main motivations for the recent extensions of the model.

An attempt to incorporate Stribeck effect into the Dahl model was done [Bliman, P.-A. and Sorine, M, 1991] by introducing a second-order Dahl model using linear space invariant descriptions. The new model, called Bliman-Sorine, describes the friction as a function of the path only. and it does not depend on how fast the system moves along the path. Expressed as a linear system in the space variable $s=\int_0^t \left|v(\tau)\right| d\tau$, the model has been included in library and written as follows:

$$(3)\quad \begin{aligned} \frac{dx_S}{ds} &= Ax_S + Bv_S \\ F &= Cx_S \end{aligned}$$

where $v_S = \operatorname{sgn} v$ and x_s is the displacement and $A=\begin{pmatrix} -1/\left(\eta\varepsilon_f\right) & 0 \\ 0 & -1/\varepsilon_f \end{pmatrix}, B=\begin{pmatrix} f_1/\left(\eta\varepsilon_f\right) \\ -f_2/\varepsilon_f \end{pmatrix}, C=\begin{pmatrix}1 & 1\end{pmatrix}$ are the second order model matrix with $f_1 - f_2$ corresponding to kinetic friction exponentially reached as $s\to\infty$, $f_1 = F_C$ and stiffness $\sigma = f_1/\varepsilon_f$. The model can be viewed as a parallel connection of a fast and a slow Dahl model. The fast model has higher steady state friction than the slow model. The force from slow model is subtracted from the fast model, which results in a stiction peak. Therefore, the friction peak mentioned above emulates quite accurately the equivalent of stiction for a dynamic model but does emulate the Stribeck effect only at a certain distance after motion starts.

The essential elements of the concepts presented so far have been lately integrated in the friction model aiming to establish a link between tribology (all mechanical aspects of contacting surfaces) and modeling for control [Canudas de Wit, C. *et al.*, 1995]. Named LuGre model of friction (abbreviation from Lund and Grenoble), the model is related to the bristle interpretation of friction. It includes Stribeck effect, rate dependent friction phenomena such as varying break-away force and frictional lag. Friction is modeled as the average deflection force of elastic springs associated to the contact. When a tangential force is applied, the bristles will deflect like springs. If the deflection is sufficiently large the bristles start to slip. The average bristle deflection (the new state of friction process - *z*) for a steady state motion is determined by velocity. It is lower at low velocities, which implies that the steady state deflection decreases with increasing velocity. The model has the form:

$$(4)\quad \frac{dz}{dt} = v - \frac{\sigma_0 \left|v\right|}{g\left(v\right)} z$$

$$(5)\quad F = \sigma_0 z + \sigma_1 \dot{z} + \alpha_2 v$$

where z is the pre-sliding displacement or, more accurately, the average deflection of the bristles, σ_0 and σ_1, are the stiffness of bristle and, respectively, the damping, α_2 is viscous friction. The function $g\left(v\right)$ is the function describing Stribeck’s effect. A parameterization proposed for $g\left(v\right)$ is given hereafter:

$$(6)\quad g\left(v\right) = F_C + \left(F_S - F_C\right) e^{-\left(v/v_S\right)^2}$$

The Bliman-Sorine (B.-S.) and the LuGre models are both extensions of the Dahl model. The dahl model has many attractive features. It is a dynamic model that captures many aspects of friction. It has, however, a serious drawback because does not describe stiction. The Bliman-Sorine and the LuGre models attempt to also capture the stiction phenomenon in two different ways: using a two Dahl models in parallel (B.-S.) and by introducing a velocity varying coefficient, σ_1 (LuGre).

3. LIBRARY CONTENT

The library of friction models developed by author as a Simulink® toolbox is presented in Figure 2 and contains the following components:

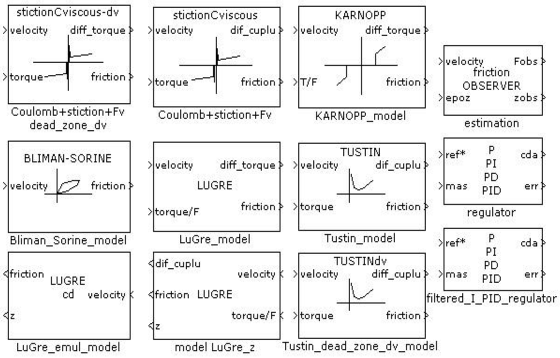


Fig.2. The Simulink® toolbox for friction model and control

- two ways approach of a generalized KFM (with and without a zero velocity interval);
- Tustin model (also in 2 approaches via *DV*);
- Karnopp model;

- Bliman-Sorine model;
- LuGre model.

Beside the friction models mentioned above, other control design components are included: a friction observer and 2 PID controllers.

4. MODELS BLOCK DIAGRAM

In this section, each representative library component will be presented like it has been implemented in Simulink®. An example of parameters block corresponding to model mask is illustrated in order to exemplify the user-friendly features of library.

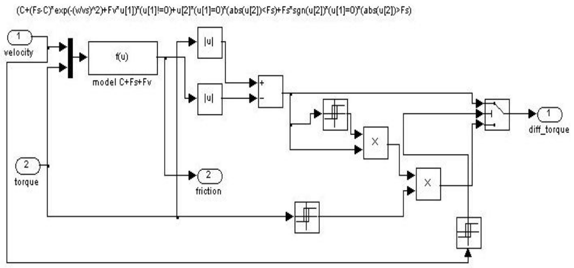


Fig.3. Generalized KFM – with DV interval

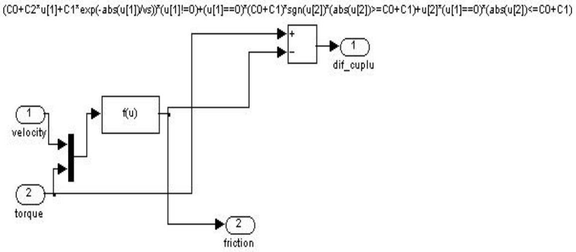


Fig.4. Tustin model

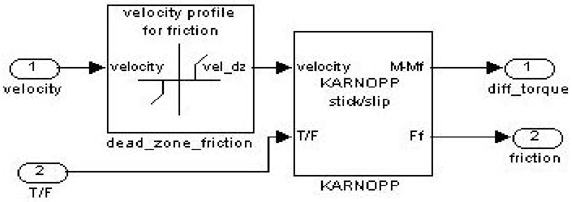


Fig.5. Karnopp model

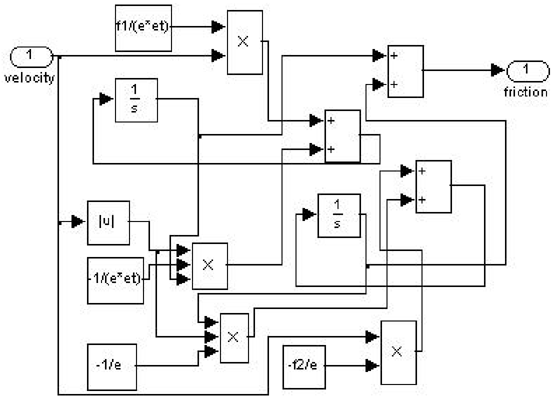


Fig.6. Bliman-Sorine model

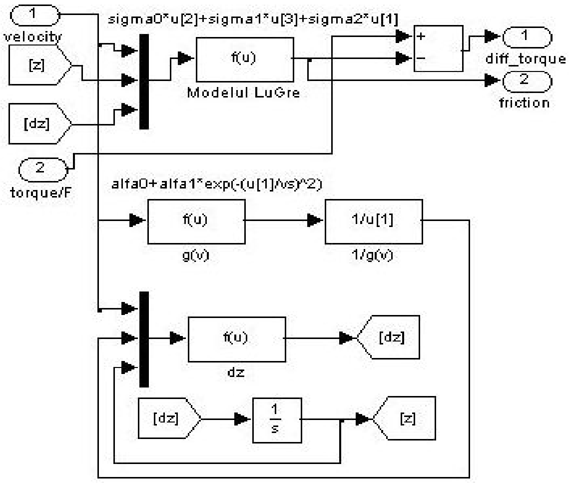
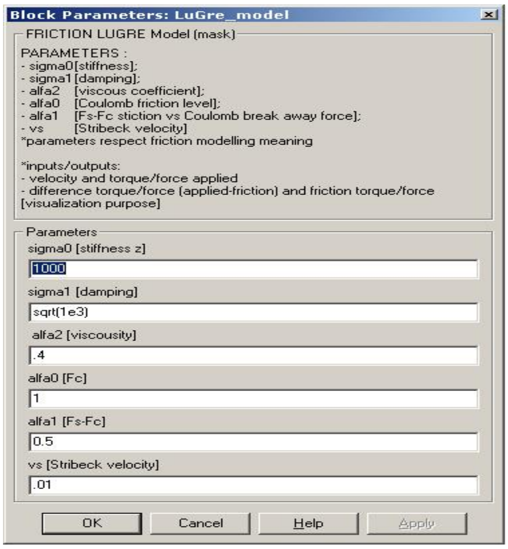


Fig.7. LuGre model



FRICTION LUGRE Model (mask)

PARAMETERS :
- sigma0[stiffness];
- sigma1 [damping];
- alfa2 [viscous coefficient];
- alfa0 [Coulomb friction level];
- alfa1 [Fs-Fc stiction vs Coulomb break away force];
- vs [Stribeck velocity]
*parameters respect friction modelling meaning

*inputs/outputs:
- velocity and torque/force applied
- difference torque/force (applied-friction) and friction torque/force [visualization purpose]

Parameters
sigma0 [stiffness z]: 1000
sigma1 [damping]: sqrt(1e3)
alfa2 [viscousity]: .4
alfa0 [Fc]: 1
alfa1 [Fs-Fc]: 0.5
vs [Stribeck velocity]: .01

Fig.8. Parameters block for LuGre model

5. SIMULATION RESULTS

The behavior for small displacements is of particularly interest for control (applications that involve precision pointing or positioning) because friction force changes very rapidly when system operates under this type of motion. The first simulation test that reveals most about behavior at small displacement (so-called pre-sliding regime) is to apply an input force $F = b + a\sin\omega t$ to an open loop simple drive system described by:

$$(7)\quad J\frac{d^2x}{dt^2} + F_{friction} = u$$

where $F_{friction}$ is given by a library model. Parameters a and b have been chosen such as to get either small velocities or velocity reversals. The results of simulation are presented in Figure 9 and Figure 10 for two different models: KFM and LuGre. Friction force is shown as a function of displacement. It can be observed that the static model does not capture the hysteresis loop such as the real laboratory experiments exhibit it.

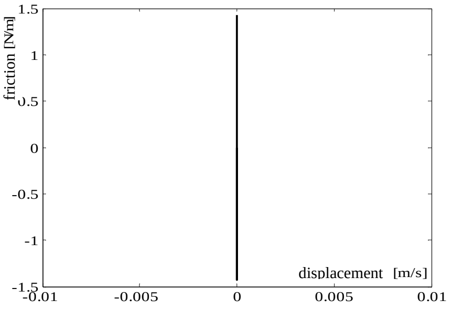


Fig.9. Pre-sliding displacement as described by generalized KFM (a=0.15 and b=0)

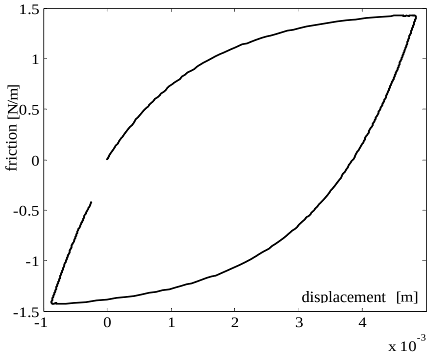


Fig.10. Pre-sliding displacement as described by LuGre friction model (a=0.15 and b=0)

The second simulation test is made to investigate stick-slip motion. A mass is attached to a spring with certain stiffness. The end of the spring is pulled with constant velocity. The results of simulation for the system mentioned above is presented in Figure 11 and 12 as a comparative outlook facing a static model (Tustin) and a dynamic model (LuGre). Both models have validated stick-slip motion, the significant notice that might be done is regarding the accuracy in reproducing real phenomenon.

The last part of current section is dedicated to illustrate a typical servo-problems and one of the control engineering solutions to eliminate it. The issues taken into account from this paper point of view were the limit cycles caused by friction and an example of model based compensation scheme for friction. Figure 13 presents the block diagram for limit cycles problem for system (7) with PID control. It has been observed that friction gives rise to limit cycles in servo drives where the controller has integral action. Only a linear feedback from position was used in the PID control law, knowledge about friction being not used. Afterwards, the same servo drive has been employed to test a model-based compensation technique. Figure 14 depicts a block diagram for the position control problem using a friction observer for friction compensation. The simulations were carried out using the LuGre library model. All characteristics are represented in latter figures.

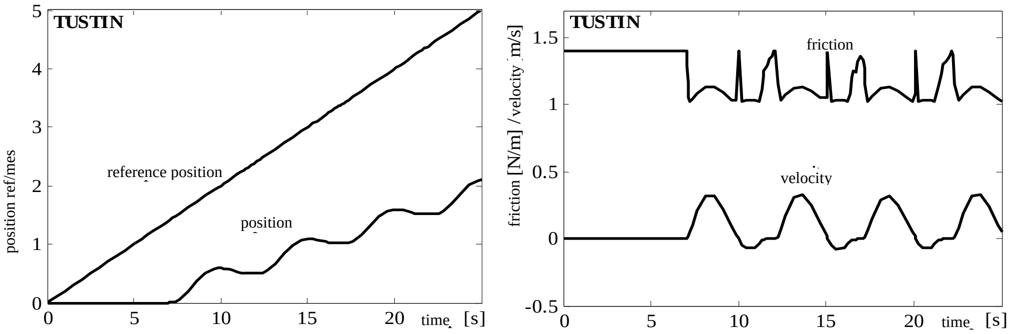


Fig.11. Simulation of stick-slip motion for Tustin model: position characteristics (left); friction and velocity (right)

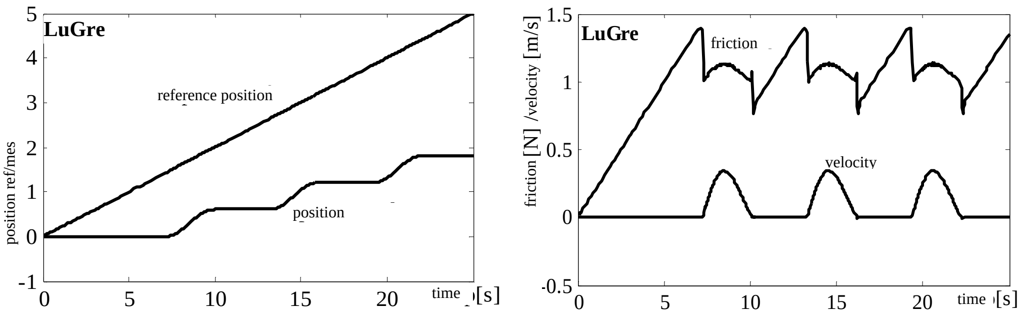


Fig.12. Simulation of stick-slip motion for LuGre model: position trajectories (left); friction and velocity (right

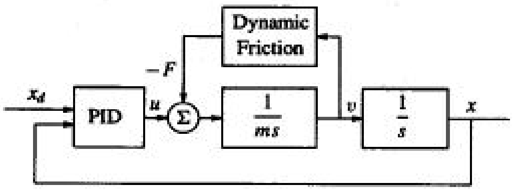


Fig.13. Block diagram for limit cycles servo problem with PID controller

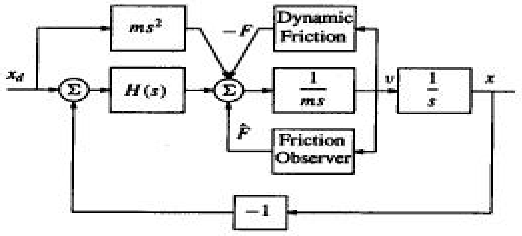


Fig.14. Block diagram for position control with friction observer

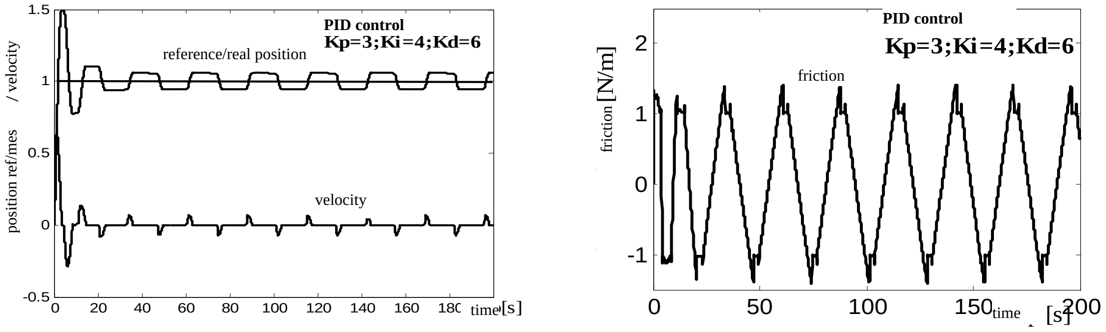


Fig.15. Simulation of PID position control limit cycles problem in Fig.13: position trajectories and speed (left); friction (right)

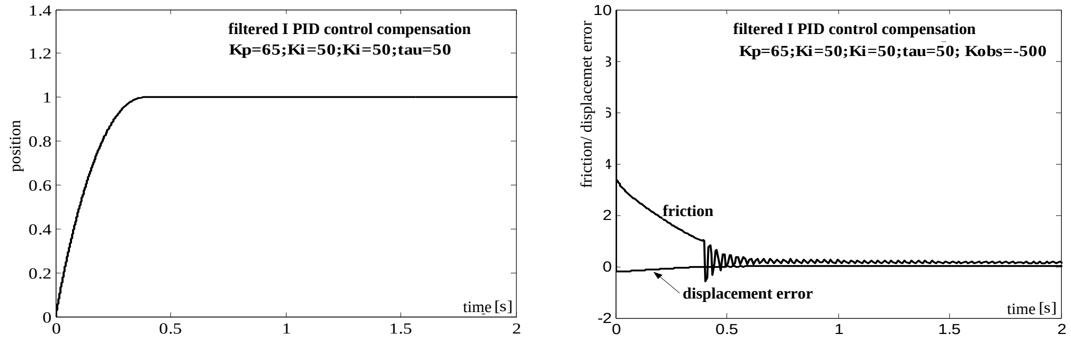


Fig.16. Simulation of model based compensation using a friction observer: position and friction characteristics

6. CONCLUSIONS

The paper presents a Simulink approach to friction phenomena simulations for both research and education purposes. A custom organized toolbox including KFM, dynamic friction models, controllers and friction observer was developed employing grouping and masking Simulink facilities.

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