The Application of Euler – Lagrange Method of Optimization for Electromechanical Motion Control

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Abstract: Industrial and non-industrial processes such as production plans, robots, pumps, compressors, home applications, transportation of people and goods etc., require some kinds of motion control. The main functions of electromechanical drives are to adjust these processes by controlling the torque, speed or position. The objective of this paper is to perform the control of motion while minimizing power losses, that is $?Ri^2dt$, in process conversion of electrical energy to mechanical energy. The optimal control laws for our problem is find using the Euler – Lagrange principle. We consider three types of controlled drives: torque, speed and position. Each of them has different control laws. By implementation of these controls with Borland C++ and Matlab environment, substantial energy savings are obtained.

Keywords: Motion control, sensor less, energy saving, ac drives system

1. INTRODUCTION

The electrical drives have a wide range of practice in industry and transportation. It is possible to distinguish three different requirements with regard to the main state control:

- Torque controlled drives
- Speed controlled drives
- Position controlled drives

Depending on the application there are different demands for accuracy or dynamic response. With increasing accuracy the cost and complexity rise steeply.

In the classical approach, the minimization index to be performed is the accuracy and the time of the dynamic response. That is, the system components must have the capability to move in minimum time with the best accuracy. But when the motion time T gets shorter acceleration increases by $1/T^2$ and consequently the current and power dissipation increase and motor overheats.

In this paper another choice of design objectives is used: perform a move within time T, while minimising the motor temperature.

There are different solutions of this problem, depending of the drive targets: torque, speed or position control.

2. DYNAMICS OF A MECHANICAL DRIVE

The equations describing the dynamic behavior of a mechanical drive with constant inertia are:

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$$(1)\quad J\frac{dw}{dt} = m(w,e,u_M,t) - m_L(w,e,u_L,t)$$

$$\frac{da}{dt} = w$$

where ω is the angular velocity and a is angular coordinate.

In order to gain better insight the motor and load torque m, m_L are considered as functions of ω,e and of the controls inputs u_M and u_L to the actuators on the motor and load side. Hence, by neglecting the dynamics of the motor and load, we arrived at a second order system described by the two state equation (1).

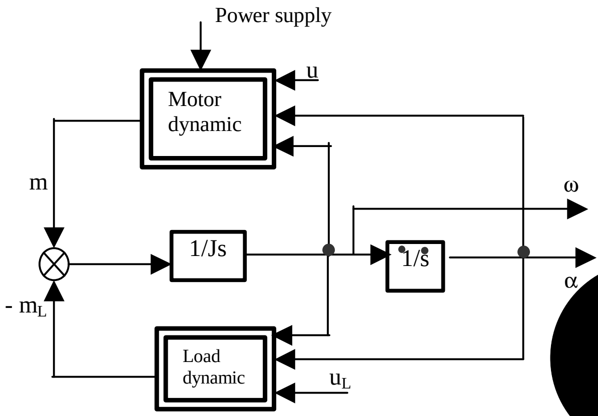

Fig.1. Block diagram for dynamics of mechanical drive

Figure 1 shows a block diagram describing the motion of the drive system. We have assumed that all moving parts of a drive can be combined to form one effective inertia J. However, for a more detailed analyses of a mechanical dynamic effects it may be necessary to consider the distribution of the masses and the linkage between then.

The two blocks with double frame describes the non linear functions and may contain additional dynamic states described by differential equations.

3. DESIGN SOLUTIONS

The well known optimal regulator design problem is to determinate the optimal control law $u^*(x,t)$ which can transfer the system from its initial state to the final state such that given performance index is minimized.

The performance index is selected to give the best trade-off between performance and cost of control. The performance index that is widely used in optimal control design is known as the quadratic performance index and is based on minimum-error and minimum-energy dissipation criteria.

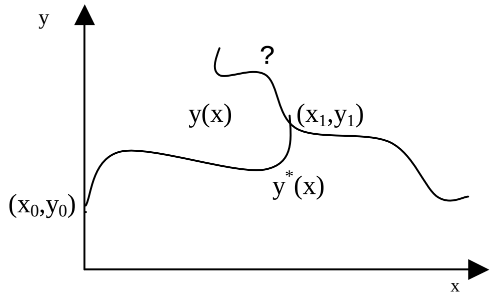

Fig.2. Two paths for transfer from (x_0, y_0) state to (x_1, y_1) state.

Consider the system described by

$$(2)\quad \dot{x}(t) = Ax(t) + Bu(t)$$

The quadratic performance index is:

$$(3)\quad I = \int_{t0}^{tf} \left(x^T Qx + u^T Ru \right) dt$$

Subject to the dynamic system equation (2). In (3) Q is a positive semi-definite matrix on R is a real symmetric matrix. Q is positive semidefinite if all its principal minors are non-negative. The choice of the [redacted]lements of Q and R allows the relative weighting of [redacted]dividual state variable and individual inputs.

[redacted]tegrals variations methods (the Potriagin's [redacted]m principle or the Hamiltonian method) for [redacted]timal control low optimization give the [redacted]ities in practical implementation, because [redacted]er initial conditions or final conditions are not completely known.

In this paper, the application of Lagrangean method of optimization to our problem in considered.

If we consider the unconstraint functional:

$$(4)\quad \mathrm{I} = \int_{x0}^{x1} g(x, y, \dot{y})dx$$

it must find $y^*(x)$ which perform the minimization of *I*.

The problem's solution can be found by means of the Euler-Lagrange equation.

$$(5)\quad \frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial g}{\partial \dot{y}}\right)$$

If the target point 1 is moving on trajectory C, the transversality condition is needed (a generalized ortogonality).

$$(6)\quad \left[\left(g - \dot{y} * g_{\dot{y}}\right)dx + g_{\dot{y}}\,dy\right]_1 = 0$$

The optimisation problem for dynamic systems based on the Euler-Lagrange principle starts from the general criteria:

$$(7)\quad I_0 = \int_{t_0}^{t_1} L_0(x(t),u(t),t)dt + M_0(x^0,t_0,x^1,t_1)$$

where L_0 and M_0 are functions defined in $X * U * t \to R^1$, respectively in $X * T \to R^1$

Usually there are three types of optimisation problem:

- The Lagrange problem, when $L_0 \neq 0$ and $M_0 = 0$;
- The Mayer problem when $L_0 = 0$ and $M_0 \neq 0$;
- The Bolza problem when $L_0 \neq 0$ and $M_0 \neq 0$;

If the general state equation of the system is:

$$(8)\quad \dot{x}(t) = f\left(x(t),u(t).t\right)$$

The function Φ is defined as:

$$(9)\qquad \Phi(x,\dot{x},u,t) = f(x,u,t) - \dot{x} = 0$$

For Lagrange problem the functional criteria defined as:

$$(10)\qquad I = L_0(x,u,t) + l^T * \Phi(x,\dot{x},u,t)$$

where λ represents the Lagrange multipliers.

The Euler-Lagrange equation for the new functional criteria are:

$$(11)\quad \begin{aligned} \frac{dI}{dx} &= \frac{d}{dt}\left(\frac{dI}{d\dot{x}}\right) \\ \frac{dI}{du} &= \frac{d}{dt}\left(\frac{dI}{d\dot{u}}\right) \\ \frac{dI}{d\,l} &= \frac{d}{dt}\left(\frac{dI}{d\,\dot{l}}\right) \end{aligned}$$

By means of Euler-Lagrange equations we can find the optimal control laws, first for mechanical motion system, and then for electrical drive system.

4. MOTION CONTROL LAWS WHICH MINIMISING THE MOTOR TEMPERATURE.

The equations describing the motions of drive with constant inertia and constant load torque are:

$$(12)\quad J\dot{w} = m - m_L$$

$$(13)\quad \begin{aligned} \dot{a} &= w \\ \dot{m}_L &= 0 \end{aligned}$$

The performance measure of energy optimisation leads to the system is:

$$(14)\quad I_0 = R\int i^2 dt\,.$$

The motion torque equation is:

$$(15)\quad m = k_m * i\,;$$

where:

- for dc motor,

$$k_m = k\,f$$

- for ac motor with field orientation

$$k_m = \frac{3}{2}\frac{f_2}{1 + s_2}$$

where ϕ and ϕ_2 are the flux of the machines.

From equations (13,14), results:

$$(16)\quad I_0 = \frac{R}{k_m^2}\int_0^T m^2 dt$$

4.1. Speed controlled drive

In this case the problem is to modify the state of the system from initial speed ω_i to final speed ω_f,with minimisation of energy dissipation in electromechanical power conversion system. In this condition the functional criteria (10) is:

$$(17)\quad I = \frac{R}{k_m^2} * m^2 + l_1\left(J\dot{w} - m + m_L\right) + l_2 m_L$$

The Euler – Lagrange equations with results from (16) are :

$$(18)\quad \begin{aligned} \frac{2R}{k_m^2} * m - l_1 &= 0 \\ J\,\dot{l}_1 &= 0 \\ \dot{l}_2 &= l_1 \end{aligned}$$

with initial and final conditions:

$t=0 \quad \omega = \omega_i\,;\ t = T \quad \omega = \omega_f$

From (17) results the optimal control :

$$(19)\quad m^* = J\frac{w_f - w_i}{T} + m_L$$

where

$$(20)\quad e = \frac{w_f - w_i}{T}$$

is the angular acceleration

The cost performance index is:

$$(21)\quad I_0^* = \frac{R}{k_m^2}[Je + m_L]^2$$

4.2. Position controlled drive:

In this case the problem is to achieve an imposed angular position a_f in time T with minimum cost.

The performance index (10) is:

$$(22)\quad I_0 = \frac{R}{k_m^2}m^2 + l_1[J\dot{w} - m + m_L] + l_2(\dot{a} - w) + l_3\dot{m}_L$$

and the Euler-Lagrange equations are:

$$(23)\quad \begin{aligned} &\frac{2R}{k_m^2}m - l_1 = 0 \\ &J\dot{l}_1 = -l_2 \\ &\dot{l}_2 = 0 \\ &\dot{l}_1 + \dot{l}_3 = 0 \end{aligned}$$

The optimal control law results from these equation by integration:

$$(24)\quad m^* = \frac{k_m^2}{2R}(C_1 - C_2 t)$$

where C_1 and C_2 are integration constants. With initial and final conditions

t=0 $m^*=m_0+m_L$, $a=0$

t=T $m^*=m_L-m_0$ $a=a_f$;

results:

$$(25)\quad \begin{aligned} m^* &= m_0\left(1 - 2\frac{t}{T}\right) \\ w &= \frac{m_0}{J}\left(t - \frac{t^2}{T}\right) - \frac{m_L t}{J} \\ a &= \frac{m_0}{J}\left(\frac{t^2}{2} - \frac{t^3}{3T}\right) - \frac{m_L t^2}{2J} \\ m_0 &= \frac{J}{T^2}6\left[a_f + \frac{m_L T^2}{2J}\right] \end{aligned}$$

The equations (25) describes the optimal motion of the system in time when the imposed movement angle is a_f. The evaluation of motion cost can be effectuated by means of the relation (15):

$$(26)\quad I_0 = \frac{R}{k_m^2}\int_0^T (m^*)^2\,dt$$

or by mechanical work calculations:

$$(27)\quad L = \int_0^{a_f} m^*\,da$$

Finally, the performance index (15) for position drive system is:

$$(28)\quad I_0^* = 12\frac{RJ^2}{k_m^2 T^3}\left(a_f + \frac{m_L T^2}{J}\right)^2$$

The optimal control laws for speed and position drives are different. In the speed drive the aims is to transfer the speed of the system from ω_I to ω_f with minimal displacement a. In the position drive, the aim is to transfer the system from initial position to finally position a_f in time T with condition: $\int \omega dt = \max$. The performance index for position controlled drive with usual control law with constant dynamic torque for speed drive is:

$$(29)\quad I_1 = 16\frac{RJ^2}{k_m^2 T^3}\left(a_f + \frac{m_L T^2}{J}\right)$$ which is much greater than optimal performance index (27).

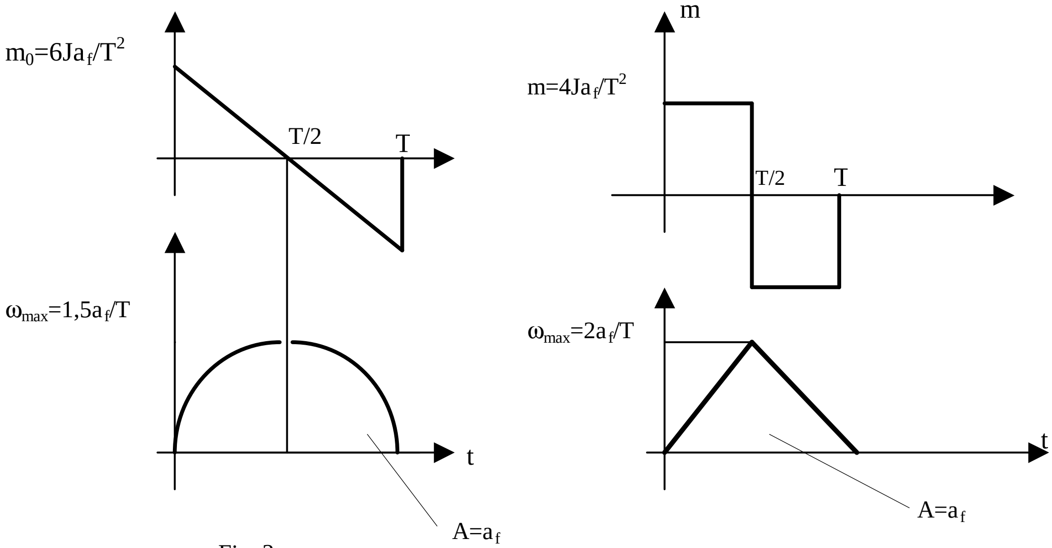

Fig. 3a

Fig. 3b

Fig. 3 Torque and speed profiles for optimal position control (a) and respectively sub optimal control (b)

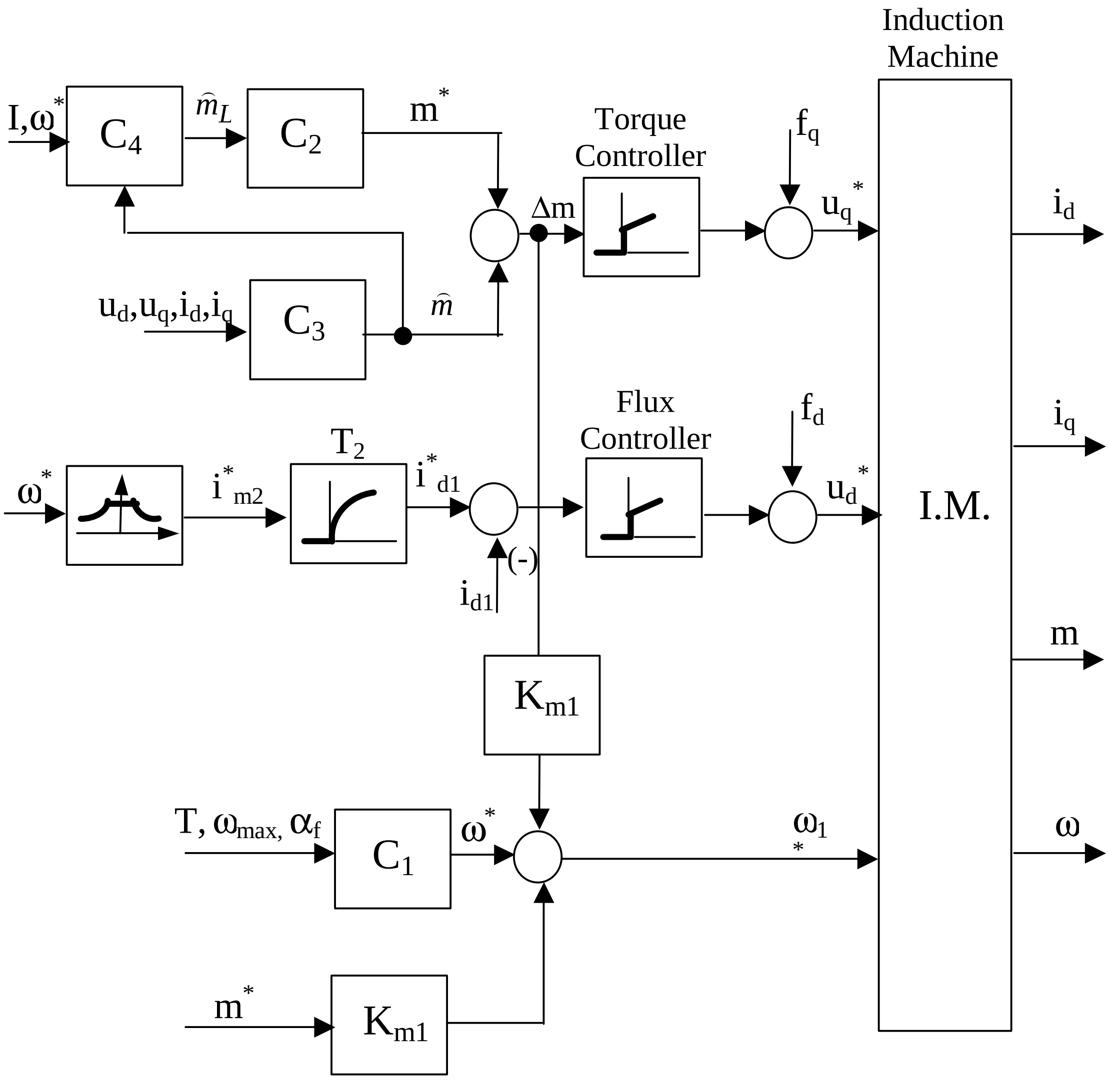

Fig.4 Block diagram of the control system for induction machine position drive system

5. EXPERIMENTAL RESULTS

We consider the position drive system with asynchronous machine. The imposed values for the system are : the time of execution T, the maximum speed w_{max} ,the final position a_f and the estimated load torque m_L. The command system calculate the optimal trajectory for speed (C1) and torque (C_2), fig.4. Then the control system find the appropriate u_d^* , u_q^* machine supply tensions components and their pulsation w_1. In figure 4 the block diagram of the control system is presented.

There are two PID controllers for torque and speed. The energy conversion process is nonlinear .For the compensation of the nonlinearites the function f_d and f_q are used. The pulsation w_1^* are calculate in the rotor field orientation theory manner, that is:

$$(30)\quad w_1^* = w + mk_{m1}$$

$$k_{m1} = \frac{3R_2}{2pf_2^2}$$

where f_2, R_2 are the rotor flux and rotor resistance, respectively .Because the drive is not sensored with speed sensor the measured speed w in (29) is replaced by the calculated speed w^* (24).

The induction motor rated parameters used for experimental tests are : 4kW,380V,1500rpm,50Hz.

In figure 5 are represented the results for the following test conditions :

$$\text{T=1 sec,}\quad w_{max} = 70s^{-1}$$

$$(31)\quad a = 46rad, m_L = 0;$$

$$(32)\quad \int i^2 dt = 1{,}9e3ws$$

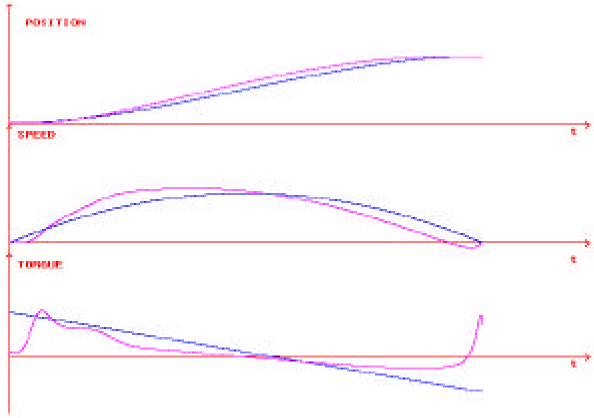

Fig.5 Imposed values and experimentals results for position speed and torque with optimal velocity profile

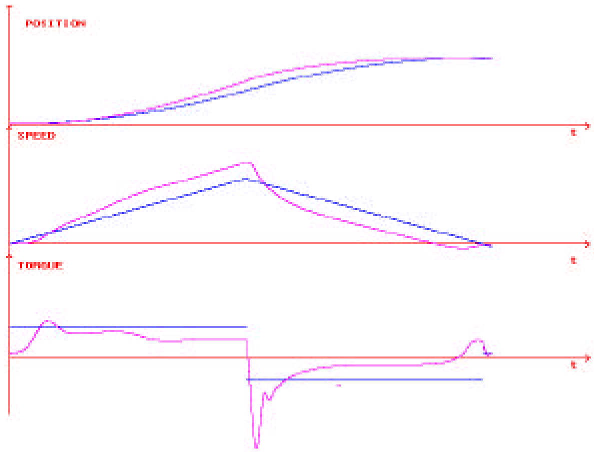

Fig.6 Imposed values and experimentals results for position speed and torque with sub optimal velocity profile

The optimal velocity profile is a parabola, which are quite well followed by the drive system without speed sensor.

If the velocity profile is triangular (fig.6)

$$\omega_{max} = 92s^{-1}$$

$$(33)\quad \int i^2 dt = 5{,}16e3ws; \alpha = 46rad$$

6. CONCLUSIONS

The control of motion with minimizing power loses in process conversion of electrical energy to mechanical by Euler-Lagrange method of optimization is a new way in the field of electrical drives.

A sensor less control system with PWM supply inverter for induction motor was developed.

In this case the imposed final position α_f is well reached without speed sensor. There are an important difference between the cost index in optimal control case and others sub optimal controls.

However the main question is the proper estimation of the load torque.

7.REFERENCES

1.Leonhard,W.,Control of Electrical Drives, Springer , 1996
2.Boldea,N.,Nasar,S.A.-Vector Control of AC Drives, CRC Press,1992
3.Bivol,I.,-Modern Drives Systems, Academica, 1999